

Undergraduate Texts in Mathematics

Editors

F. W. Gehring

P. R. Halmos

Advisory Board

C. DePrima

I. Herstein

J. Kiefer

Wendell Fleming

Functions of Several Variables

2nd Edition



Springer-Verlag

New York Heidelberg Berlin

Wendell Fleming

Brown University
Department of Mathematics
Providence, Rhode Island 02912

Editorial Board

F. W. Gehring

University of Michigan
Department of Mathematics
Ann Arbor, Michigan 48104

P. R. Halmos

University of California
Department of Mathematics
Santa Barbara, California 93106

AMS Subject Classifications: 26-01, 28-01, 58-01

Library of Congress Cataloging in Publication Data

Fleming, Wendell Helms, 1928-

Functions of several variables.

(Undergraduate texts in mathematics)

Bibliography: p.

Includes index.

1. Functions of several real variables.

I. Title

QA331.F63 1977 515'.84 76-40029

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Softcover reprint of the hardcover 2nd edition 1977

ISBN 978-1-4684-9463-1

ISBN 978-1-4684-9461-7 (eBook)

DOI 10.1007/978-1-4684-9461-7

To Flo

Preface

The purpose of this book is to give a systematic development of differential and integral calculus for functions of several variables. The traditional topics from advanced calculus are included: maxima and minima, chain rule, implicit function theorem, multiple integrals, divergence and Stokes's theorems, and so on. However, the treatment differs in several important respects from the traditional one. Vector notation is used throughout, and the distinction is maintained between n -dimensional euclidean space E^n and its dual. The elements of the Lebesgue theory of integrals are given. In place of the traditional vector analysis in E^3 , we introduce exterior algebra and the calculus of exterior differential forms. The formulas of vector analysis then become special cases of formulas about differential forms and integrals over manifolds lying in E^n .

The book is suitable for a one-year course at the advanced undergraduate level. By omitting certain chapters, a one semester course can be based on it. For instance, if the students already have a good knowledge of partial differentiation and the elementary topology of E^n , then substantial parts of Chapters 4, 5, 7, and 8 can be covered in a semester. Some knowledge of linear algebra is presumed. However, results from linear algebra are reviewed as needed (in some cases without proof).

A number of changes have been made in the first edition. Many of these were suggested by classroom experience. A new Chapter 2 on elementary topology has been added. Additional physical applications—to thermodynamics and classical mechanics—have been added in Chapters 6 and 8. Different proofs, perhaps easier for the beginner, have been given for two main theorems (the Inverse Function Theorem and the Divergence Theorem.)

Preface

The author is indebted to many colleagues and students at Brown University for their valuable suggestions. Particular thanks are due Hildegard Kneisel, Scott Shenker, and Joseph Silverman for their excellent help in preparing this edition.

Wendell H. Fleming

Providence, Rhode Island
June, 1976

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