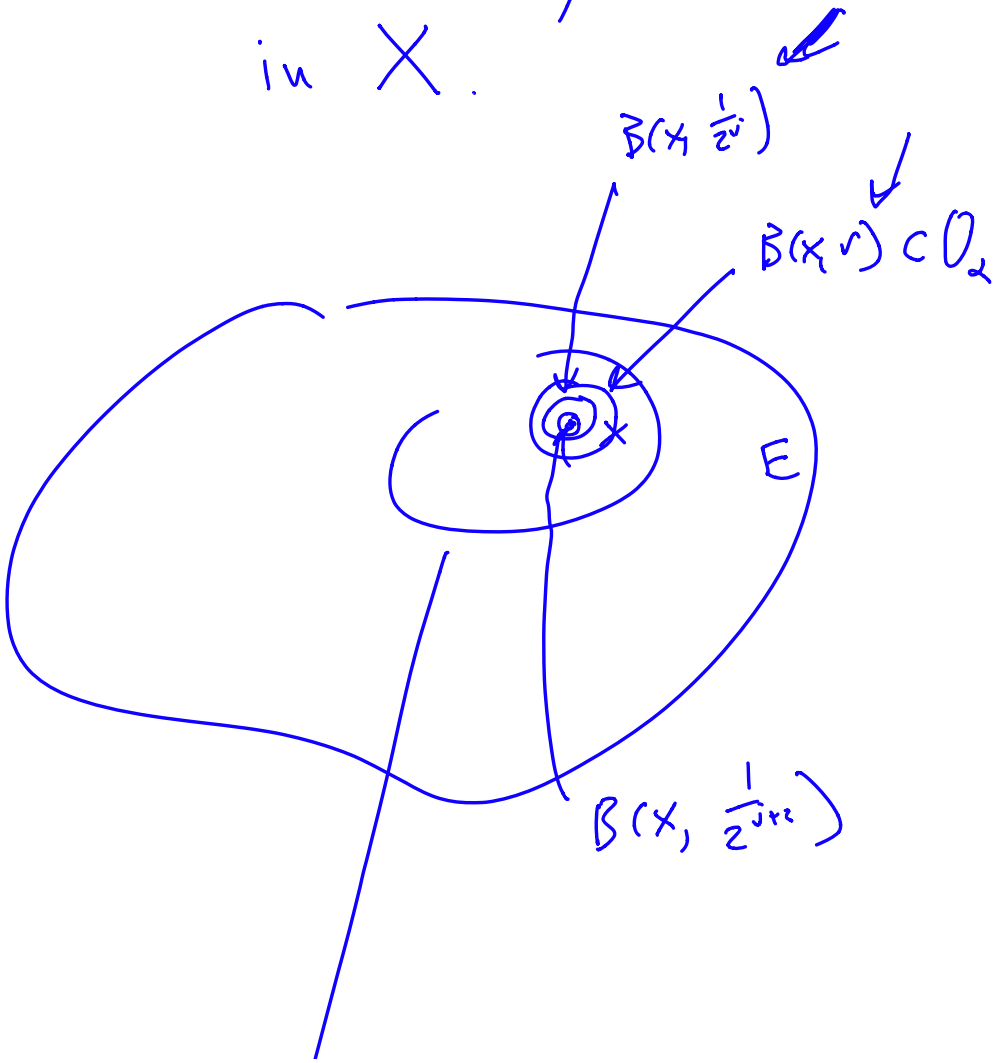
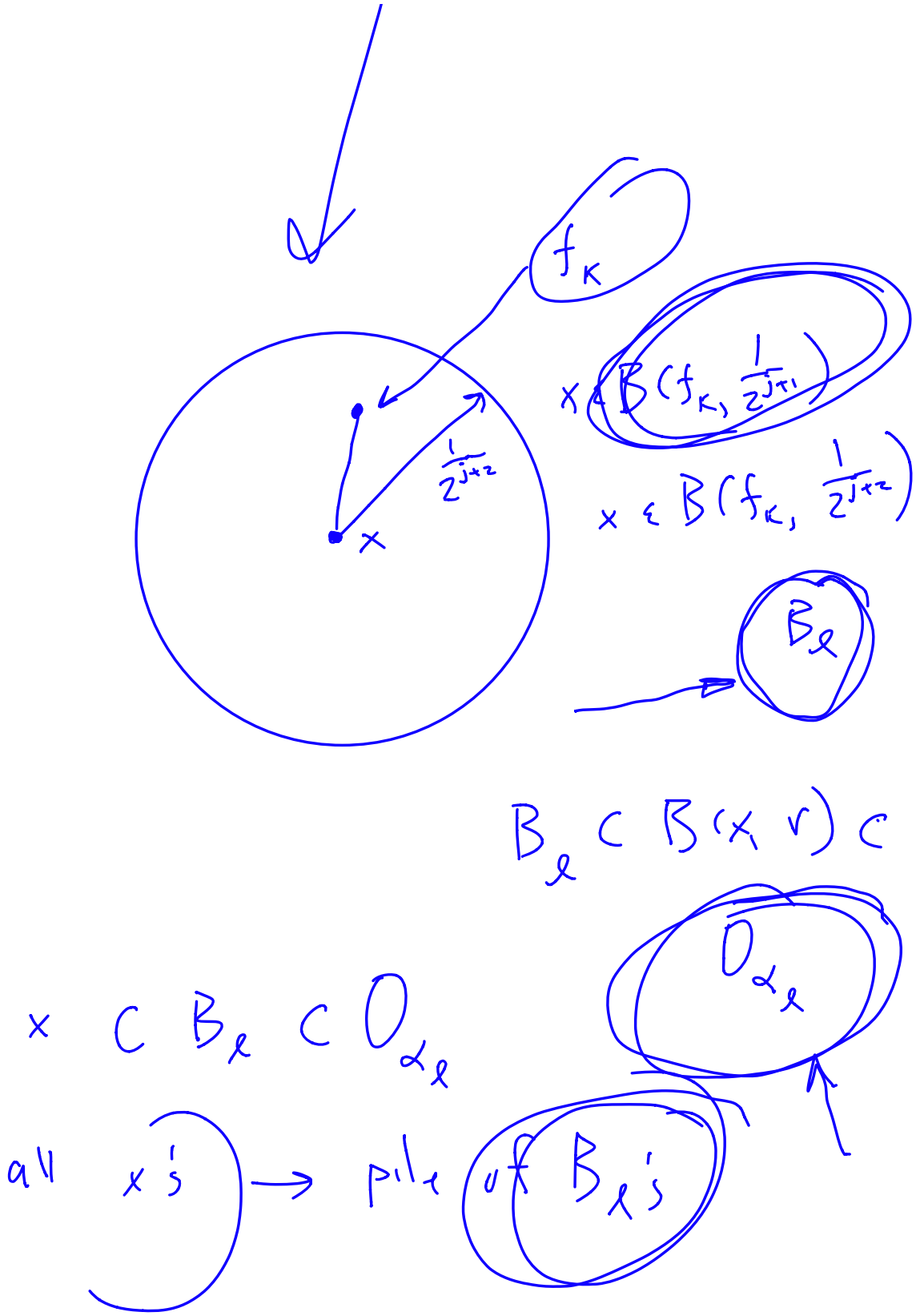
(1) — (5) refer to notes from last time.

(1) – (4) prep for (5)

F countable, dense in X.




f_K
$x \in B(f_K, \frac{1}{2^{j+1}})$
$\frac{1}{2^{j+2}}$
x
$x \in B(f_K, \frac{1}{2^{j+2}})$
B_ℓ
$B_\ell \subset B(x, r) \subset$
O_{α_ℓ}
$x \subset B_\ell \subset O_{\alpha_\ell}$
all x's
pick of B_ℓ's

Ex.

(1) prove that the rational numbers in $\mathbb{R}$, $\mathbb{Q}$ are a countable set.

(2) prove that $\mathbb{Q}$ is dense in $\mathbb{R}$ ... i.e. prove that for any $x \in \mathbb{R}$ and any $\varepsilon > 0$ $\exists$ $q_i \in \mathbb{Q}$ $\ni$

$|x - q_i| < \varepsilon$ $\quad 3.1415q\ldots$

Separability $\equiv$ there is a countable dense subset