- Metric Spaces
- Inequalities } work!

instinctive ⇓ GA easier!

Croftons formula

- various surprises from geometric Analysis } sightseeing

- other } soapbox, preaching

example: "No time to think" by David Levy

- CGAD course website up
  - notes (these notes)
  - misc links

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① Study groups ← ask for groups to form or you to ask for help

② Help session / office hours

- Jacob Strupel
- Katrina Sabochick
- Enrique Alvarado
- Adebo Sijuwade

③ I will call on you!

Metric Spaces

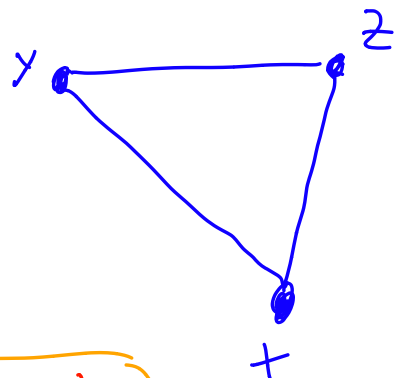
$$X \to \mathbb{R}^2, \mathbb{R}^3, G=(V,E), \ldots$$

metric $\rho : X \times X \to [0, \infty)$

① $\rho(x,y) \geq 0 \quad \rho(x,y) = 0 \iff x = y$

② $\rho(x,y) = \rho(y,x)$

③ $\rho(x,z) \leq \rho(x,y) + \rho(y,z)$



$X = \mathbb{R}^2$, $\rho(x,y) = |x-y|_2$

$$= \sqrt{(x-y)\cdot(x-y)}$$

$$= \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$$

metric?

this being 0 $\iff x = y$

$$|x| = \sqrt{x \cdot x}$$

immediate that ② $\rho(x,y) = \rho(y,x)$

① $\rho(x,y) \geq 0$ $\quad \rho(x,y) = 0 \Leftrightarrow x=y$

$$\rho(x,z) \leq \rho(x,y) + \rho(y,z)$$

$$|\underbrace{x-z}_{u+v}| \leq |\underbrace{x-y}_{u}| + |\underbrace{y-z}_{v}|$$

$$\boxed{|u+v| \leq |u| + |v|}$$

goal

$$\sqrt{(u+v)\cdot(u+v)} \leq |u| + |v|$$

$$(u+v)\cdot(u+v) \overset{?}{\leq} (|u| + |v|)^2$$

$$\underline{u\cdot u} + 2u\cdot v + \underline{v\cdot v} \overset{?}{\leq} \underline{|u|^2} + 2|u||v| + \underline{|v|^2}$$

$$2u\cdot v \overset{?}{\leq} 2|u||v|$$

$$u\cdot v \leq |u||v|$$

$$|u-v|^2 \geq 0$$

$$(u-v)\cdot(u-v) \geq 0$$

$$u\cdot u + v\cdot v - 2u\cdot v \geq 0$$

$$|u| = |v| = 1$$

$$u\cdot u + v\cdot v \geq 2u\cdot v$$

$$2 \geq 2u\cdot v$$

$$|u||v| = 1 \geq u\cdot v$$

$$u, v \longrightarrow \frac{u}{|u|}, \frac{v}{|v|}$$

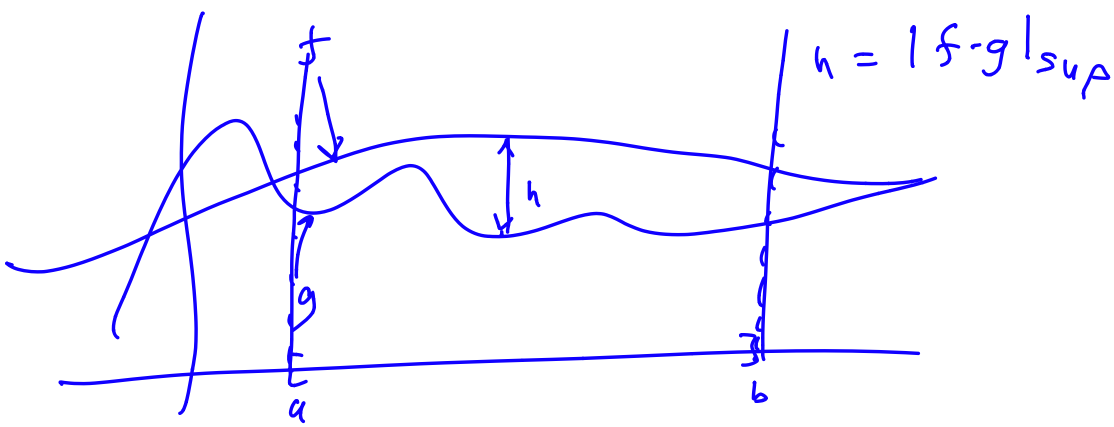
$$\frac{u}{|u|}\cdot\frac{v}{|v|} \leq 1$$

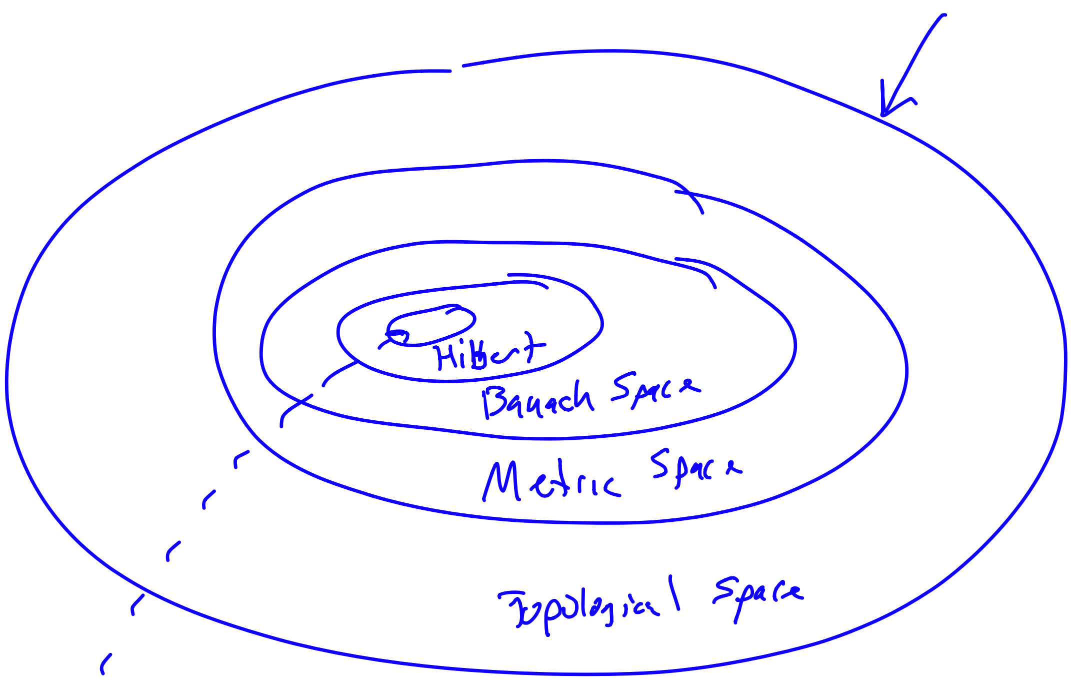
$$\boxed{u\cdot v \leq |u|\,|v|}$$

Metric Space : (X, ρ)

$\longrightarrow \mathbb{R}^n$: $(\mathbb{R}^n, |\cdot|_2)$

$C([a,b])$: metric $|\cdot|_{sup}$





Finite dimensional Hilbert Space

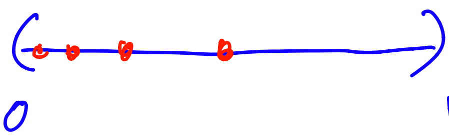
$(X, \mathcal{O})$

space is a vector space

norm

$(X, \rho = |x-y|, \text{complete})$

$(X, |x-y|, \text{complete}, |x-y| = \sqrt{\langle x-y, x-y\rangle})$



$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \cdots \frac{1}{2^n}$

finite Dimensional