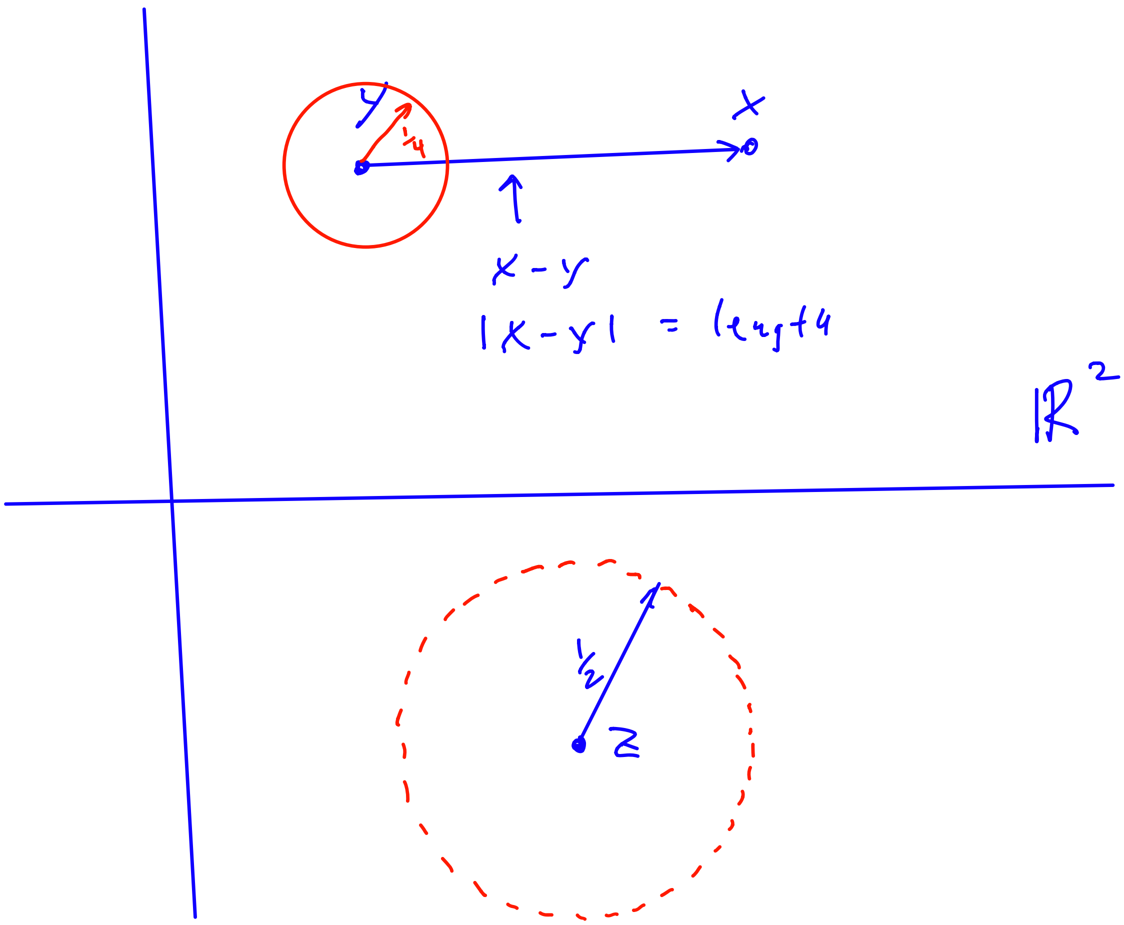


$\rho(x, y) = |x - y|$

$B(z, \tfrac{1}{2}) = \{ x \in \mathbb{R}^2 \mid |x - z| < \tfrac{1}{2} \}$

$\overline{B}(y, \tfrac{1}{4}) = \{ x \in \mathbb{R}^2 \mid |x - y| \leq \tfrac{1}{4} \}$

Def 1: (open ball)

$\varepsilon > 0$

$$B(x,\varepsilon) \equiv \{ y \in M \mid \rho(x,y) < \varepsilon \}$$

"There exists"

Def 2: (open set)

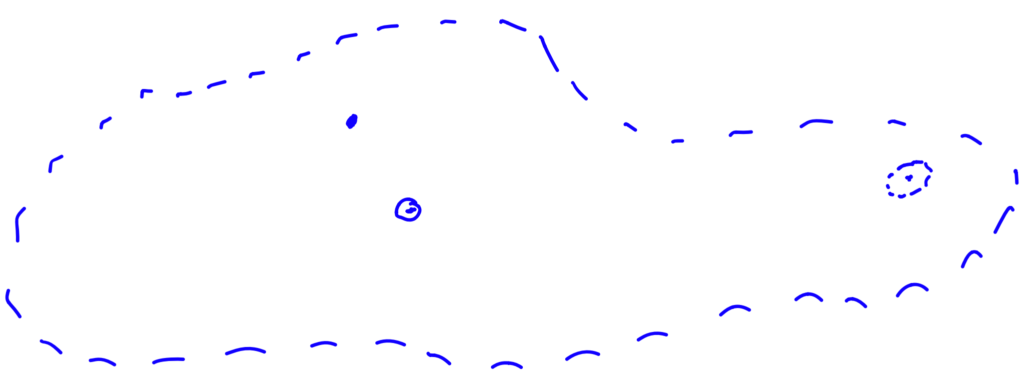
E is an open set, E is open,

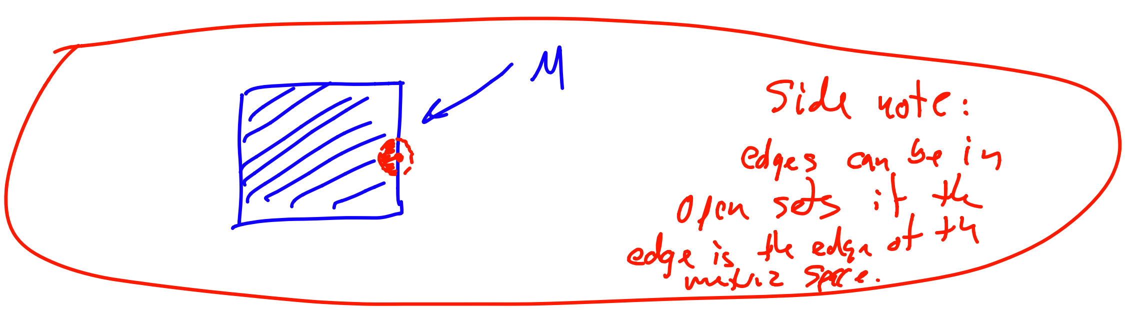
"for all"

if $\forall$ $x \in E$ $\exists$ $\varepsilon > 0$ $\ni$

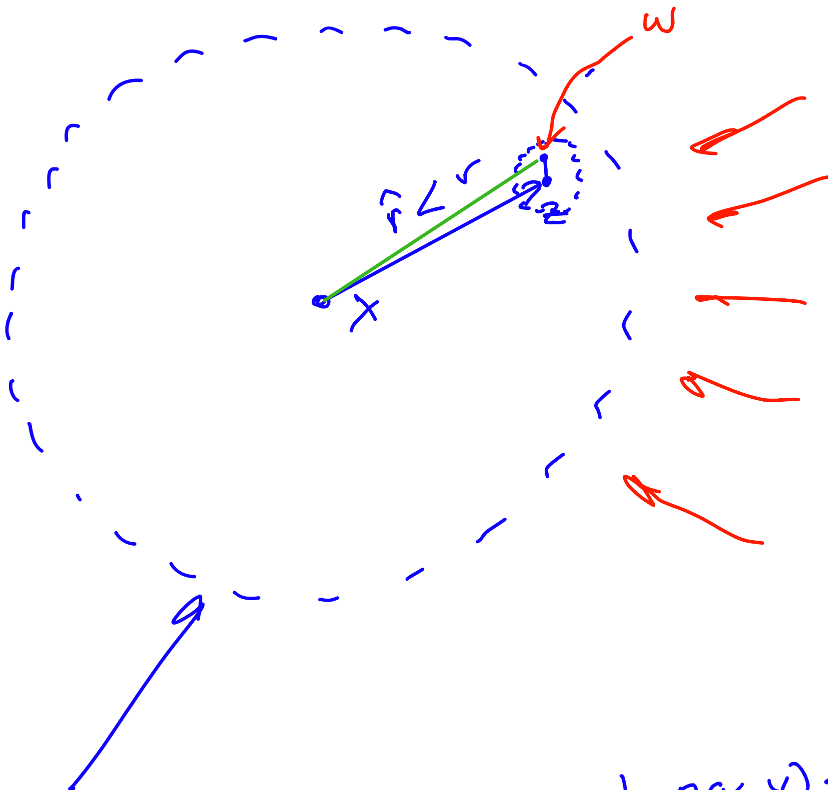
$B(x,\varepsilon) \subset E$

"Such that"





Exercise: Prove any open ball is an open set.



$$B(x, r) = \{y \in M \mid \rho(x,y) < r\}$$

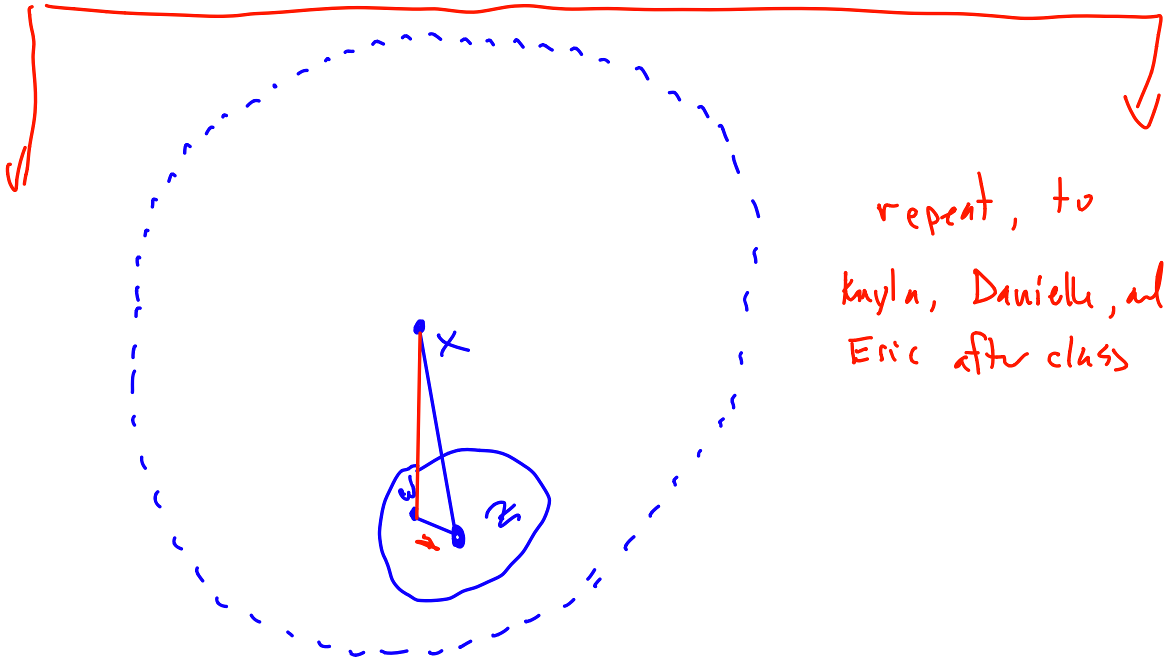
$$\rho(x,z) = \hat{r} = r - \delta \qquad \delta > 0$$

$$w \in B(z, \delta/2) \Rightarrow w \in B(x, r)$$

$$\rho(x, w) \le \rho(x, z) + \rho(z, w)$$

$\rho(x,w)$: $< r$; $\rho(x,z) = r - \delta$; $\rho(z,w) < \frac{\delta}{2}$

$$= r - \frac{\delta}{2} < r$$



repeat, to Kayla, Danielle, and Eric after class

$z \in B(x, r)$

$\rho(x, z) < r$

$\rho(x, z) = r - \delta \qquad \delta > 0$

$w \in B(z, \delta/2)$

$\left(\rho(x, w) < r \quad \text{goal!} \right)$

$\rho(x, w) \le \rho(x, z) + \rho(z, w)$

$r - \delta \qquad \le \frac{\delta}{2}$

$r - \frac{\delta}{2} < r$