

$$\rho(x, y) = |x - y|$$

$$B(z, \frac{1}{2}) = \{x \in \mathbb{R}^2 \mid |x - z| < \frac{1}{2}\}$$

$$\bar{B}(y, \frac{1}{4}) = \{x \in \mathbb{R}^2 \mid |x - y| \leq \frac{1}{4}\}$$

Def 1: (open ball)

$$\varepsilon > 0$$

$$B(x, \varepsilon) \equiv \{y \in M \mid \rho(x, y) < \varepsilon\}$$

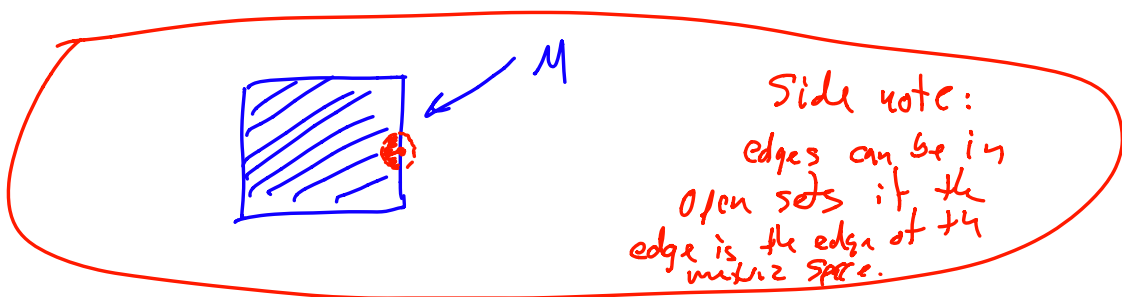
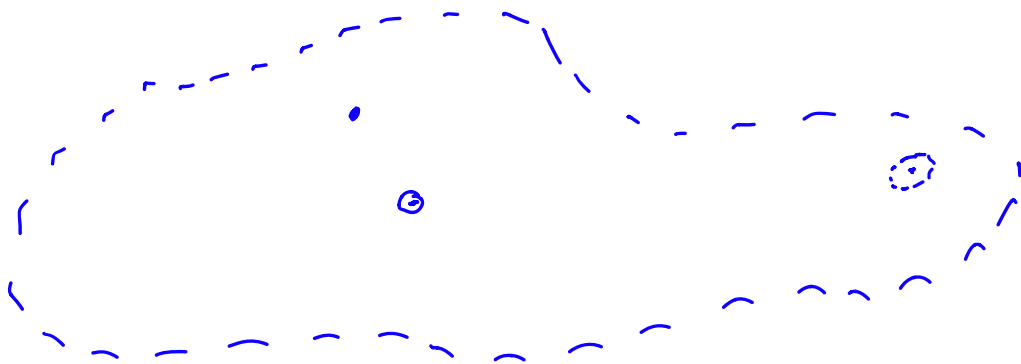
Def 2: (open set)

"for all" E is an open set, E is open,
if $\forall x \in E \exists \varepsilon > 0 \ni$

$$B(x, \varepsilon) \subset E$$

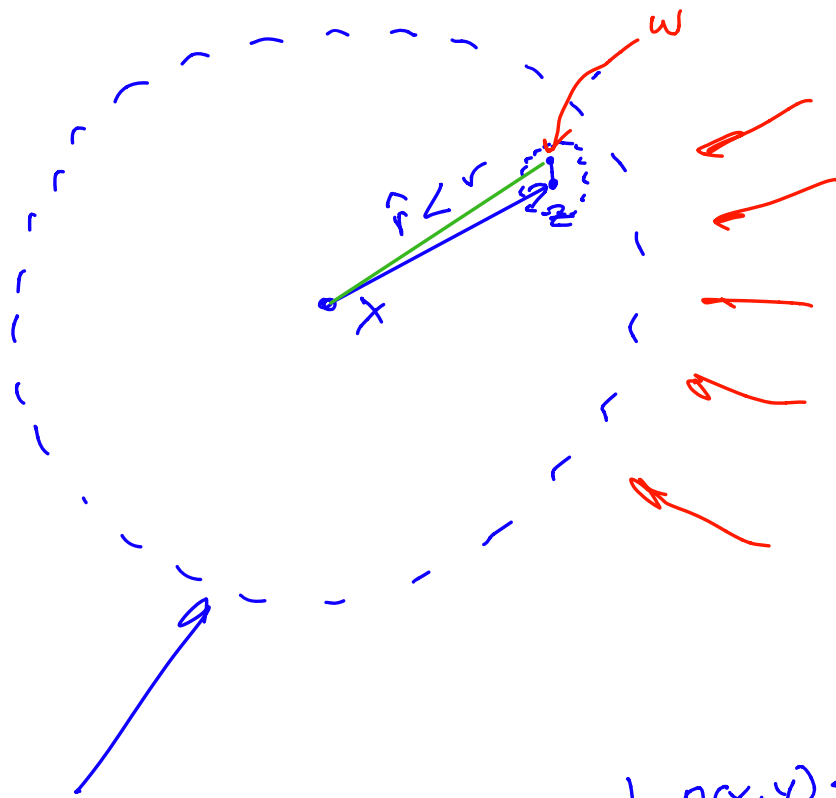
"There exists"

"Such that"



Side note:
edges can be in
open sets if the
edge is the edge of the
metric space.

Exercise: Prove any open ball is an open set.

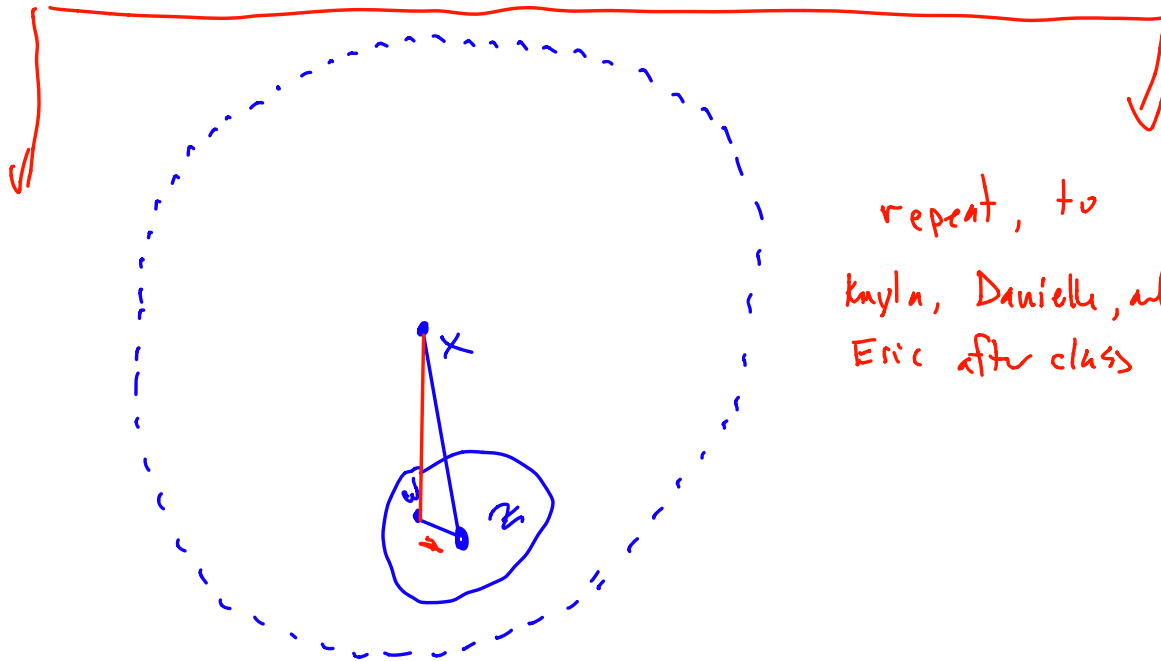


$$B(x, r) = \{y \in M \mid \rho(x, y) < r\}$$

$$\rho(x, z) = \hat{r} = r - \delta \quad \delta > 0$$

$$w \in B(z, \frac{\delta}{2}) \Rightarrow w \in B(x, r)$$

$$\begin{aligned} \rho(x, w) &\leq \rho(x, z) + \rho(z, w) \\ &\leq \overset{r-\delta}{\rho(x, z)} + \overset{\frac{\delta}{2}}{\rho(z, w)} \\ &= r - \frac{\delta}{2} < r \end{aligned}$$



repeat, to
Kayla, Daniella, and
Eric after class

$$z \in B(x, r)$$

$$\rho(x, z) < r$$

$$\rho(x, z) = r - \delta \quad \delta > 0$$

$$w \in B(z, \frac{\delta}{2})$$

($\rho(x, w) < r$ goal!)

$$\rho(x, w) \leq \rho(x, z) + \rho(z, w)$$

$$r - \delta$$

$$\leq \frac{\delta}{2}$$

$$\underline{r - \frac{\delta}{2} < r}$$