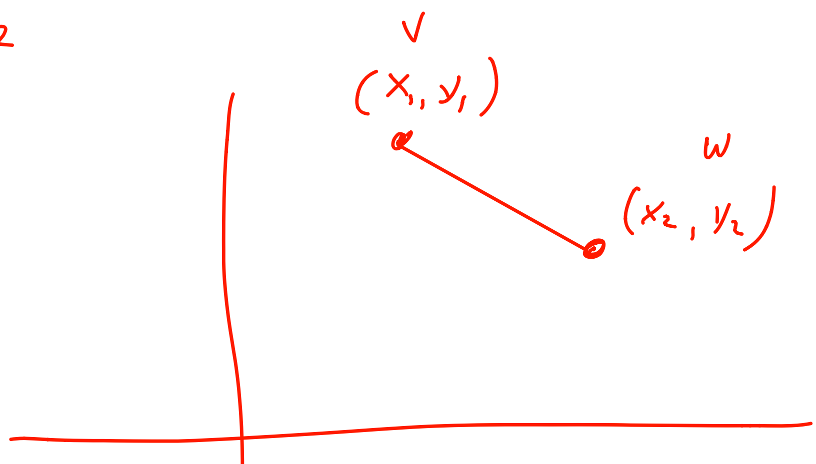
$$\int |fg| \le |f|_p \, |g|_q \qquad \frac{1}{p} + \frac{1}{q} = 1$$

$\mathbb{R}^2$



$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$= |v - w|_2$

$= \|v - w\|$

$= |v - w|$

$$\sqrt{(v - w) \cdot (v - w)}$$

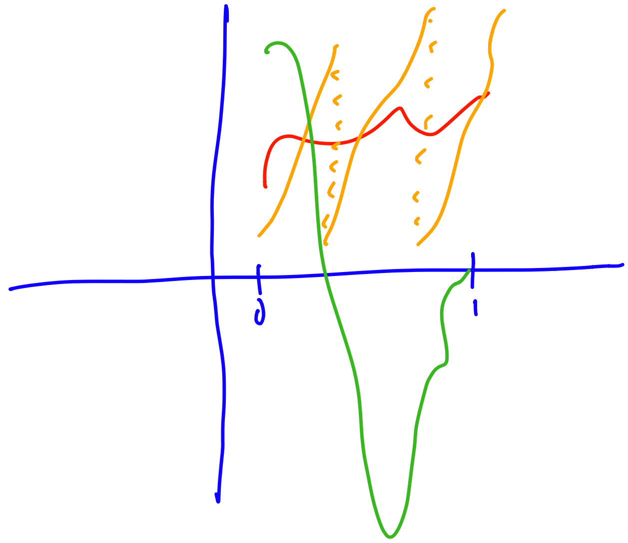
$$\sqrt{\langle v - w, v - w \rangle}$$

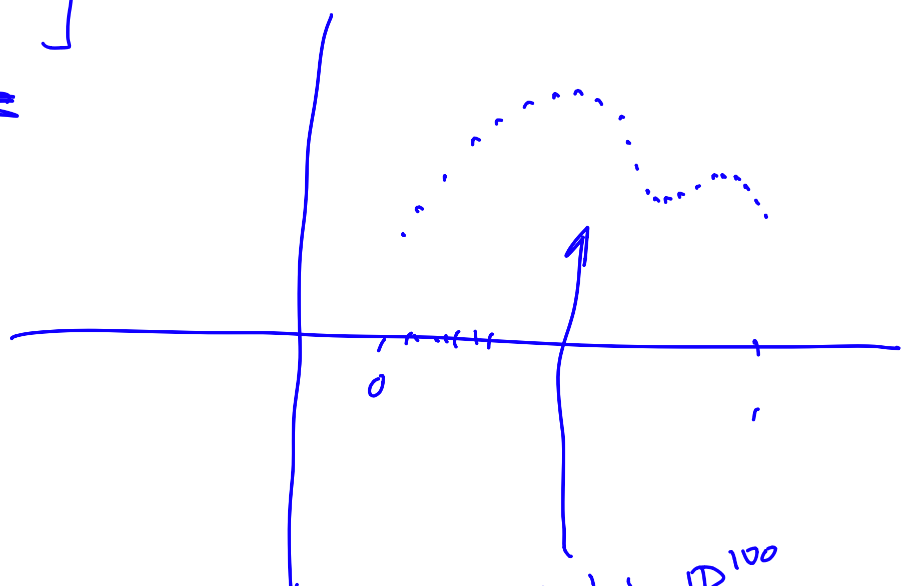
$f : [0,1] \to \mathbb{R}$

$|f|$

$\|f\|$

$$\|f\| = \left[\int_0^1 |f(x)|^2 \, dx\right]^{1/2}$$





point in $\mathbb{R}^{100}$

$$F = \big(f(0.01), f(0.02), f(0.03), \ldots f(1.00) \big)$$

$|F| = \sqrt{\text{sum of squares}}$

$$\sum_{k=1}^{100} f^2(\Delta x \cdot k)\,\Delta x \cong \int_0^1 f^2\,dx$$

$$\left(\int |f|^p\,dx \right)^{1/p}$$

$1 \le p \le \infty$

$p = 1$

$p = 2$

$p = \infty$

$p = 1 \qquad \int |f|\,dx$

$p = 2 \qquad \left[\int |f|^2\,dx \right]^{\frac{1}{2}}$

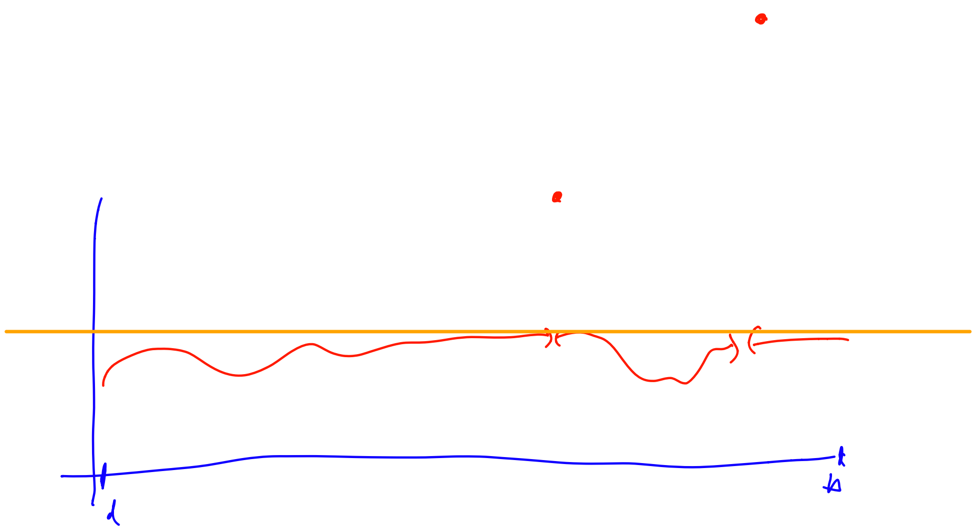
$p = \infty$ essential supremum

= smallest real number s such that the measure of the points $x \ni f(x) > s$ is 0.

i.e.

$\|f\|_\infty$ = smallest real $\ni$

$$\mu(\{x \mid f(x) > \|f\|_\infty\}) = 0$$



$\mathbb{R}^2$

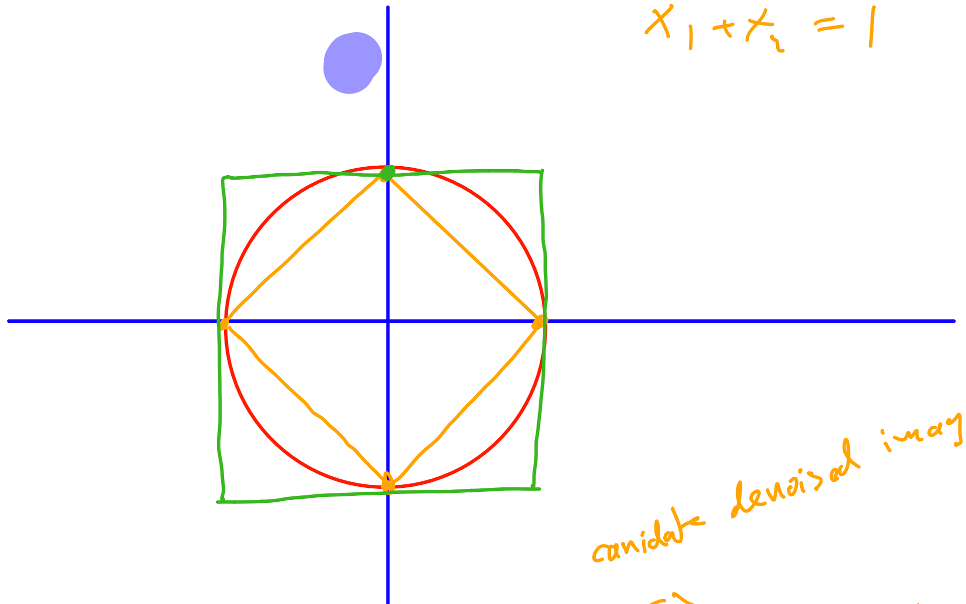
$$|x|_p \equiv \left(|x_1|^p + |x_2|^p\right)^{1/p}$$

$p = 1 \Rightarrow |x_1| + |x_2|$

$p = 2 \Rightarrow \left(|x_1|^2 + |x_2|^2\right)^{1/2}$

$p = \infty \Rightarrow \max(|x_1|, |x_2|)$

$x_1 + x_2 = 1$

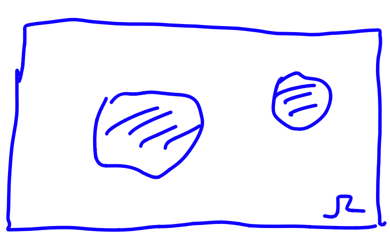


canidate denoised image

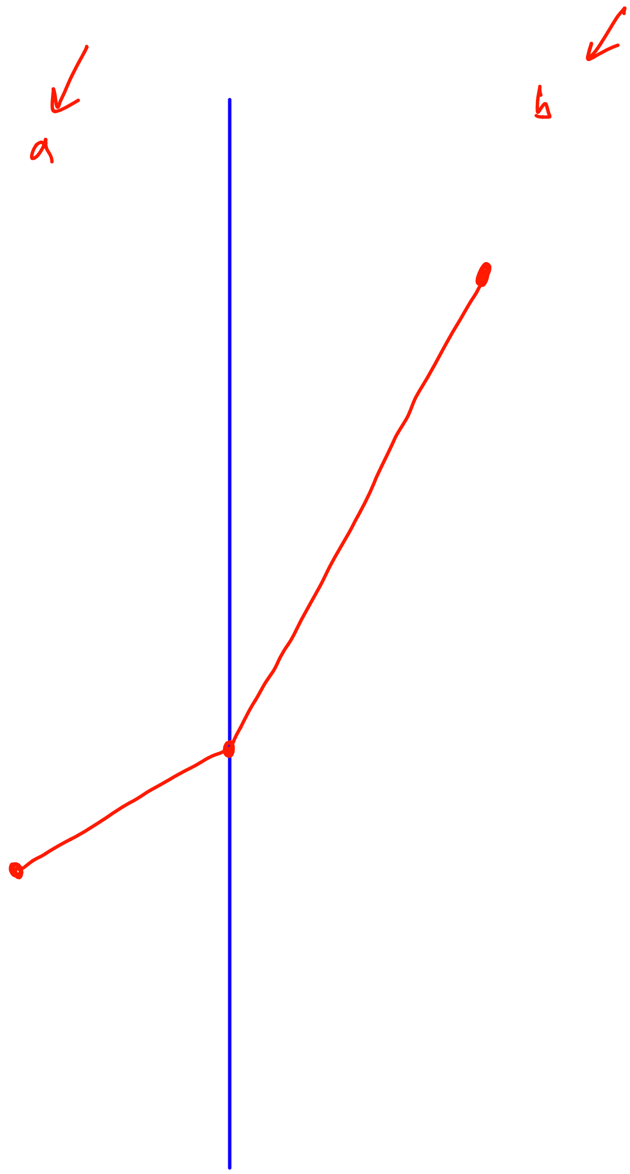
noisy measurement

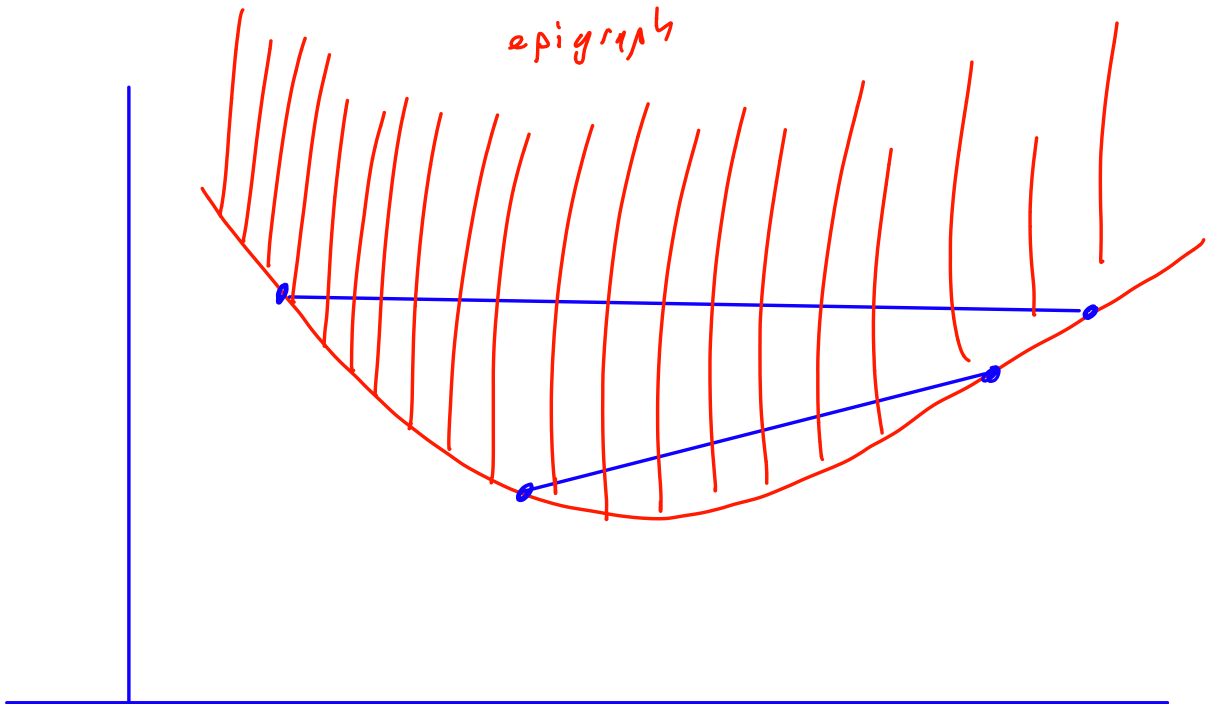
$$\min_u F_{ROF}(u) = \int_\Omega |\nabla u| \, dx + \lambda \int_\Omega |u-f|^2 \, dx$$

$$\min_u F_{CE}(u) = \int_\Omega |\nabla u| \, dx + \lambda \int_\Omega |u-f| \, dx$$



$f : \Omega \to \mathbb{R}$

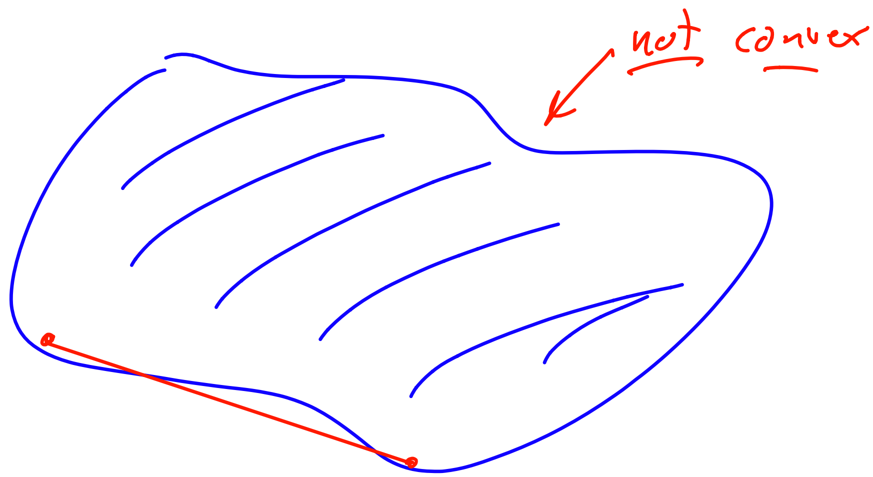

a
b



$$\boxed{\begin{array}{l} f(\alpha x+\beta y) \leq \alpha f(x) + \beta f(y) \\ \alpha+\beta=1 \end{array}}$$

f is convex $\Longleftrightarrow$ epigraph of f is a convex set

E is convex if the line segment joining any pair of points in E is also in E



f is lower semicontinuous $\iff$ $\text{epi}(f)$ is closed