

① figure 29 5.2



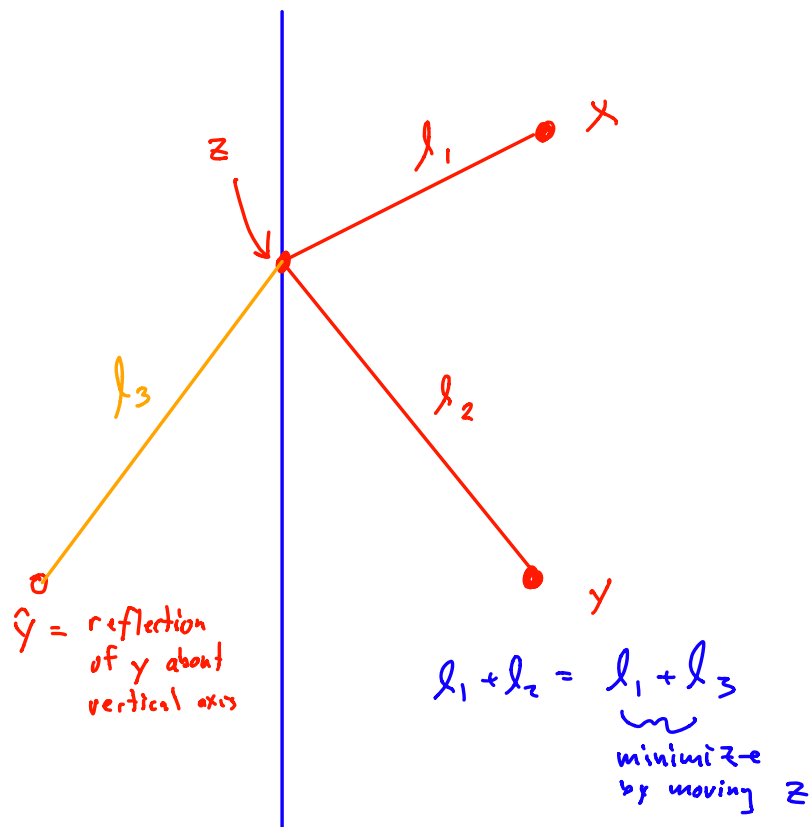
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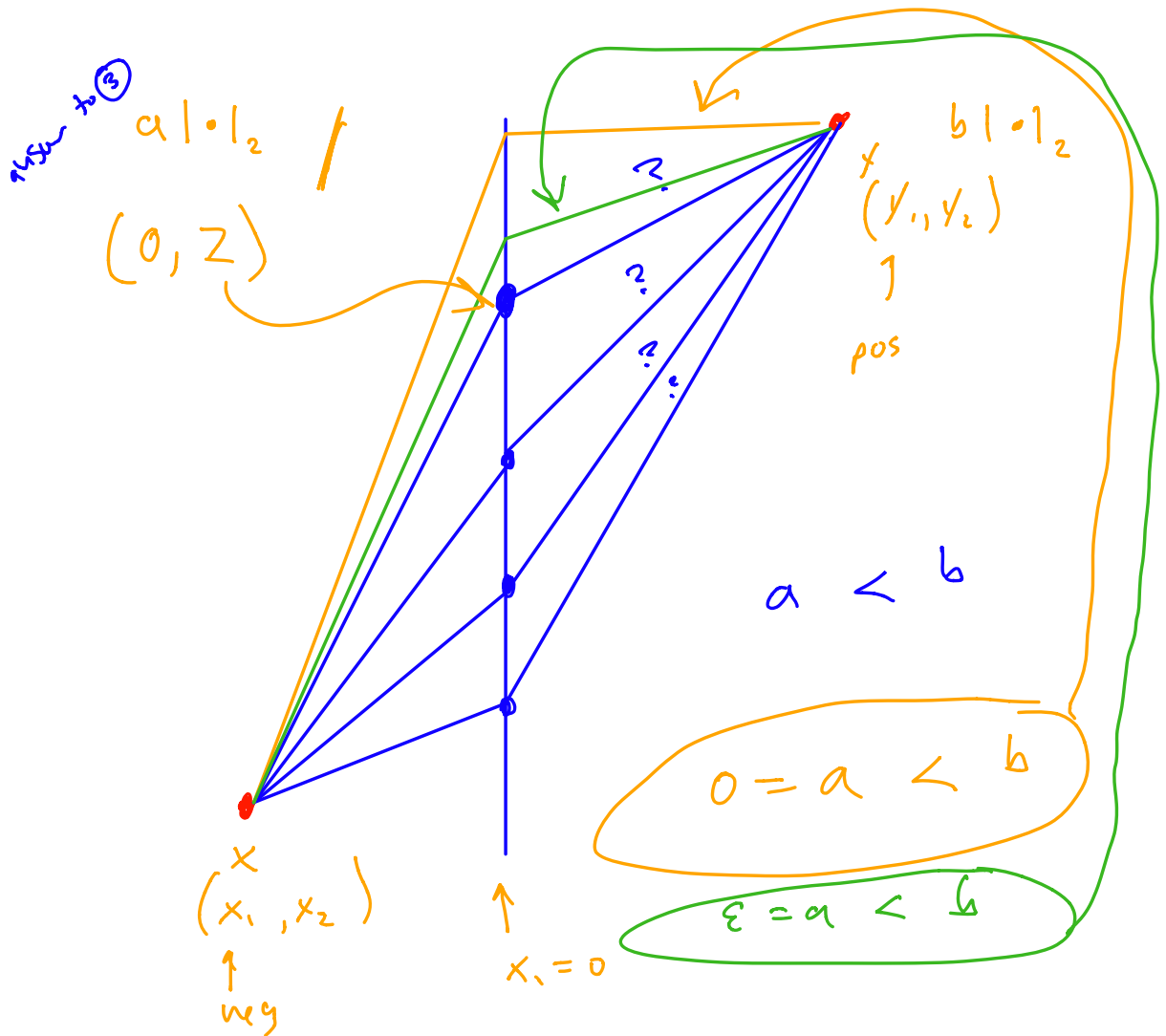
②  $|\langle \cdot, \cdot \rangle|$  ? 5.2.2 102

③ figure 30

101

answer to ③





$$\text{length} = a \left( x_1^2 + (x_2 - z)^2 \right)^{\frac{1}{2}} + b \left( y_1^2 + (y_2 - z)^2 \right)^{\frac{1}{2}}$$

$f(z)$

Compute  $\frac{d}{dz} f(z) \Rightarrow$  set = 0  
 and solve for  $z$

Figure 2

$$|x - y|$$

$$\text{e.g. } x, y \in \mathbb{R}^3$$

$$= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

Euclidean Norm

generally,

given an inner product  $\langle \cdot, \cdot \rangle$

$$x, y \rightarrow x \cdot y$$

(all inner products on finite dimensional vector spaces can be written

$$x \cdot y = x^T A y \quad \text{where}$$

$A$  is symmetric and positive definite (all eigenvalues  $> 0$ )

$$\|x\| \equiv \sqrt{\langle x, x \rangle}$$

$$\text{Euclidean norm} = \sqrt{x \cdot x}$$

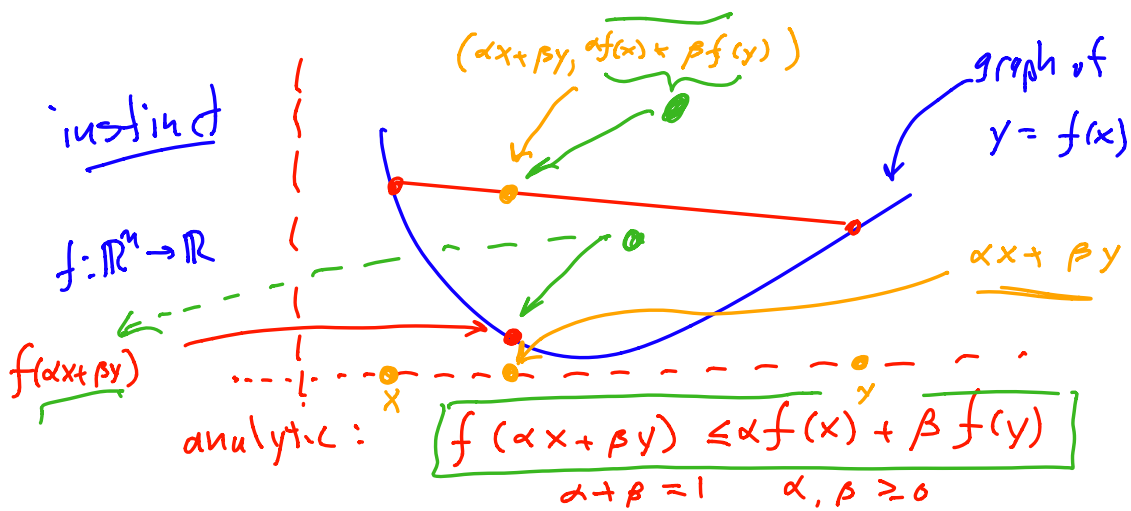
$$= \sqrt{x_1^2 + x_2^2 + x_3^2} \quad (\text{e.g. in } \mathbb{R}^3)$$

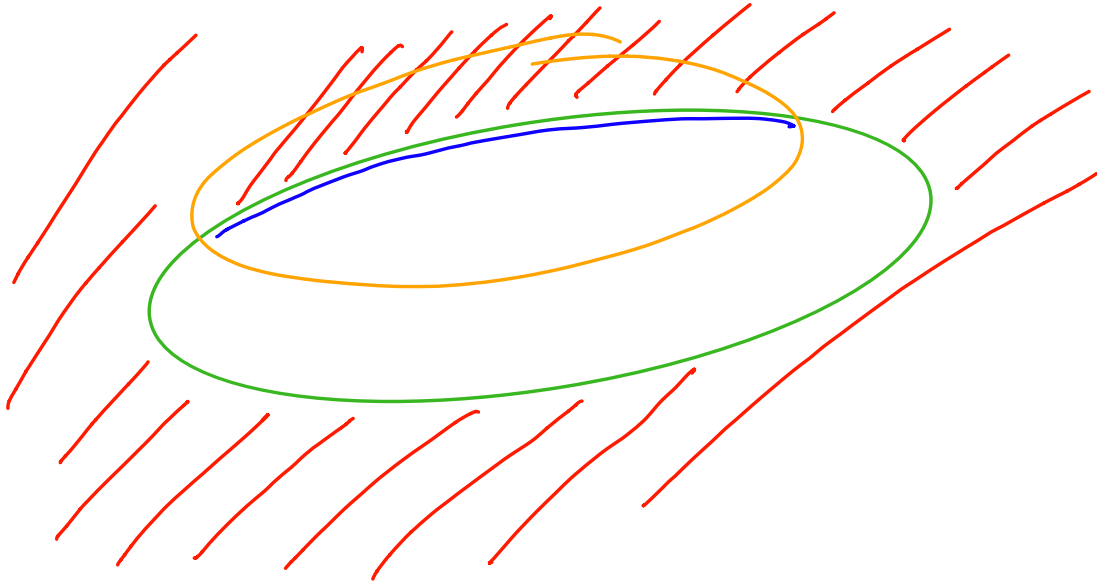
$\underbrace{\hspace{10em}}_{\geq 0}$

④ convex functions ?

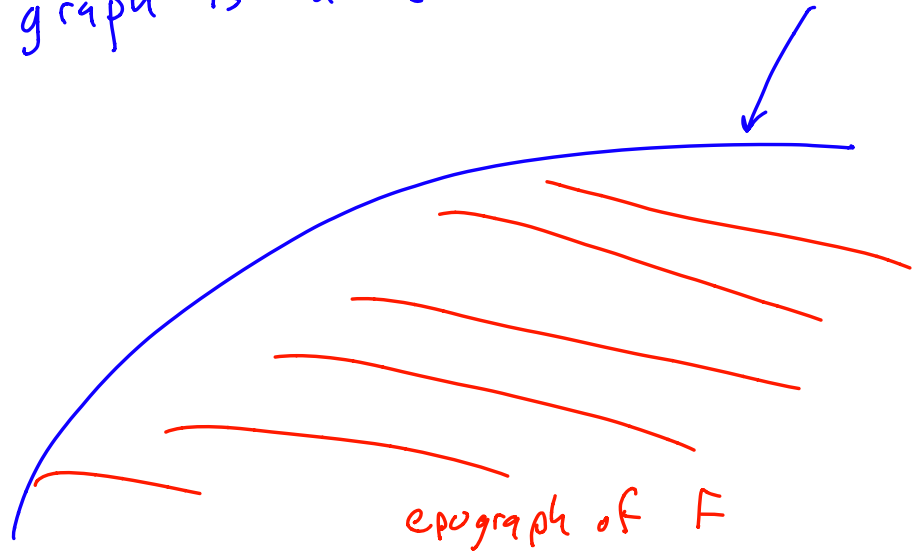
⑤ convex, concave ?

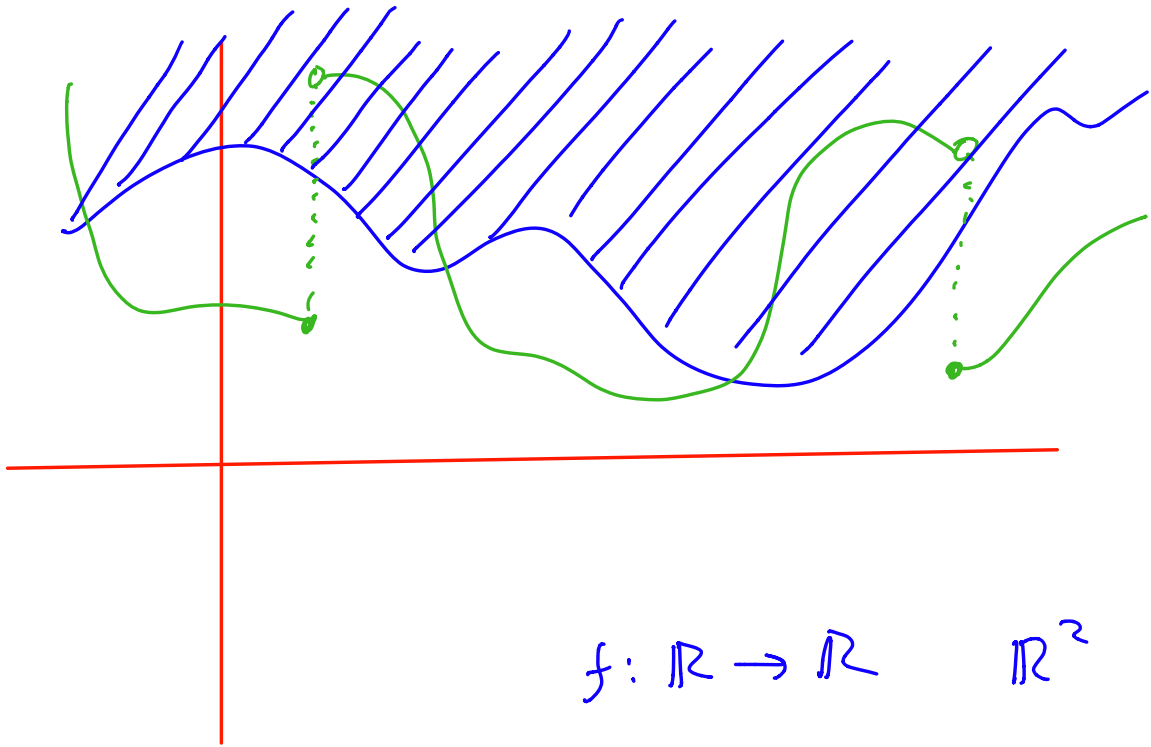
⑥ epigraph ... more examples





$F$  is a concave function if its epigraph is a convex set





$$f: \mathbb{R} \rightarrow \mathbb{R} \quad \mathbb{R}^2$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad \mathbb{R}^3$$

$$f: \mathbb{R}^4 \rightarrow \mathbb{R}^3 \quad \mathbb{R}^7$$

$$\mathbb{R}^4 \times \mathbb{R}^3$$

epigraph  $f: \mathbb{R}^n \rightarrow \mathbb{R}$

$$\equiv \{(x, y) \in \mathbb{R}^n \times \mathbb{R} \mid y \geq f(x)\}$$