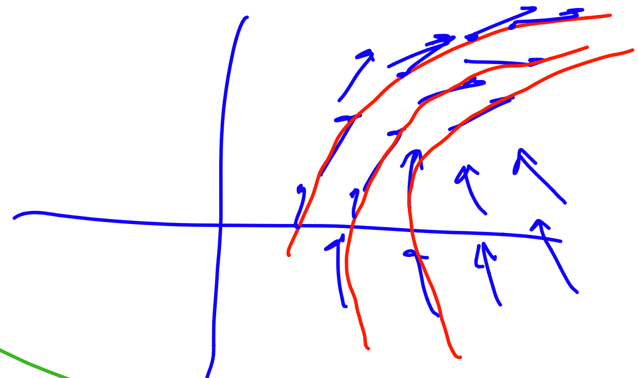
$\mathbb{R}^n$

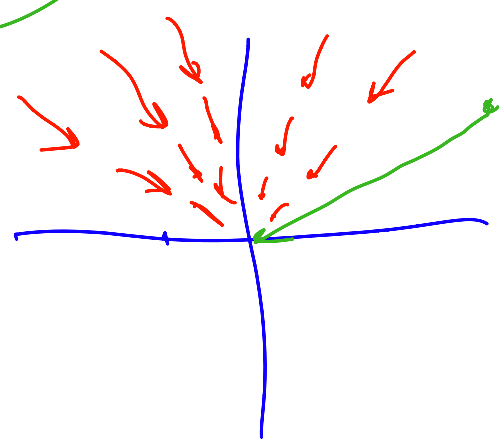
$$\underline{\underline{\dot{x} = f(x)}}$$

$f : \mathbb{R}^n \to \mathbb{R}^n$



$\underline{\| X \| < C}$

as long as $\| X(0) \| < B$



$\dot{x} = -x$

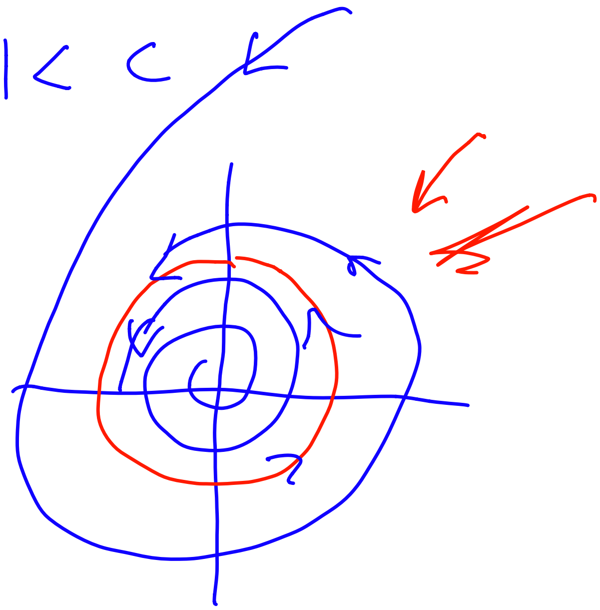
vector

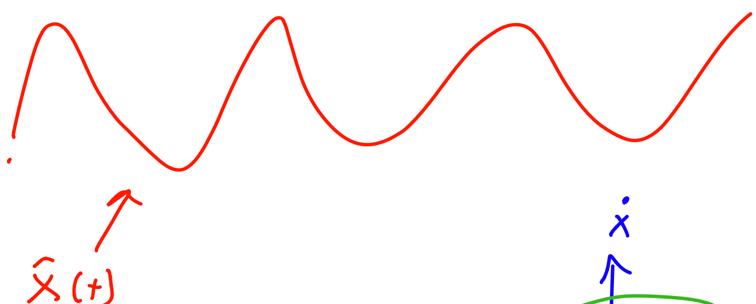
$x(t) = x_0 e^{-t}$

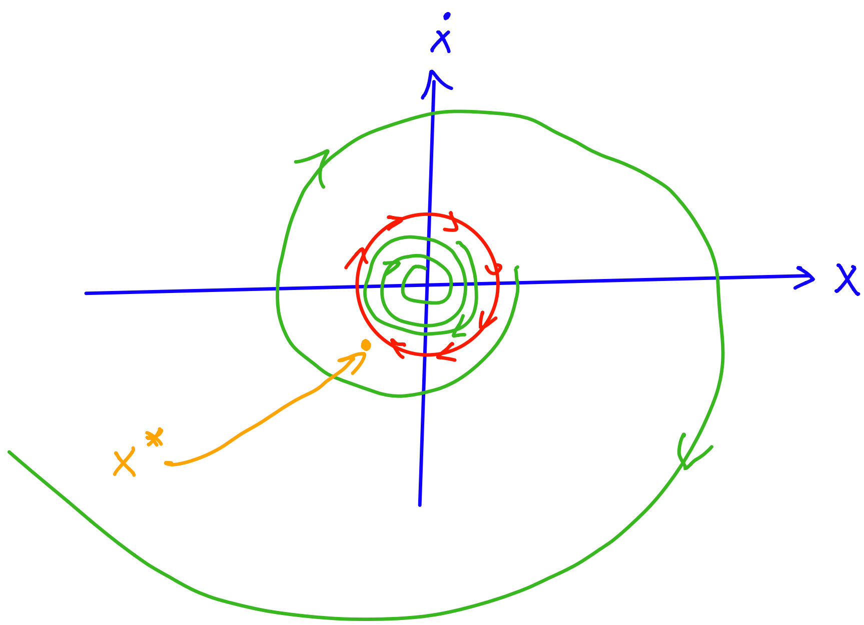
$t = 0 \; \ldots = 1$

$$\|x_0\| < c$$

$$\Rightarrow \quad \|x(t)\| < c$$





suppose $\ddot{x} = f(x)$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1)$$

$\vec{x} = (x_1, x_2)$

$\dot{\vec{x}} = \vec{f}(x)$

$\hat{x}(t)$

what is the solution to

$$\underbrace{x(t) = x^* \text{ at } t = t_0}_{\bar{x}(t)}$$

$$\|\hat{x}(t) - \bar{x}(t)\| \to 0$$
$$t \to \infty$$

suppose $\hat{x}(t)$ solves $\dot{\vec{x}} = f(x)$

with $\hat{x}(0) = x_0$

define $\hat{x}^{\varepsilon}(t)$ solution of

$\dot{\vec{x}} = f(x)$ $\hat{x}^{\varepsilon}(0) = \underline{x_0 +} \varepsilon$


$B(x_0, \varepsilon)$

1) does the difference grow?

does $\| \hat{x}(t) - \hat{x}^{\varepsilon}(t) \|$ get bigger?

2) does

$$\| \vec{x}(t) - \hat{x}^{\varepsilon}(t) \| \to 0$$

$$t \to \infty$$

1) $\| \hat{x}(t) - \hat{x}^{\varepsilon}(t) \| \leq \varepsilon \quad \forall t$

for any $\delta > 0$

2) $\| \hat{x}(t) - \hat{x}^{\varepsilon}(t) \| \leq \delta$ if

$t > N(\delta)$

---