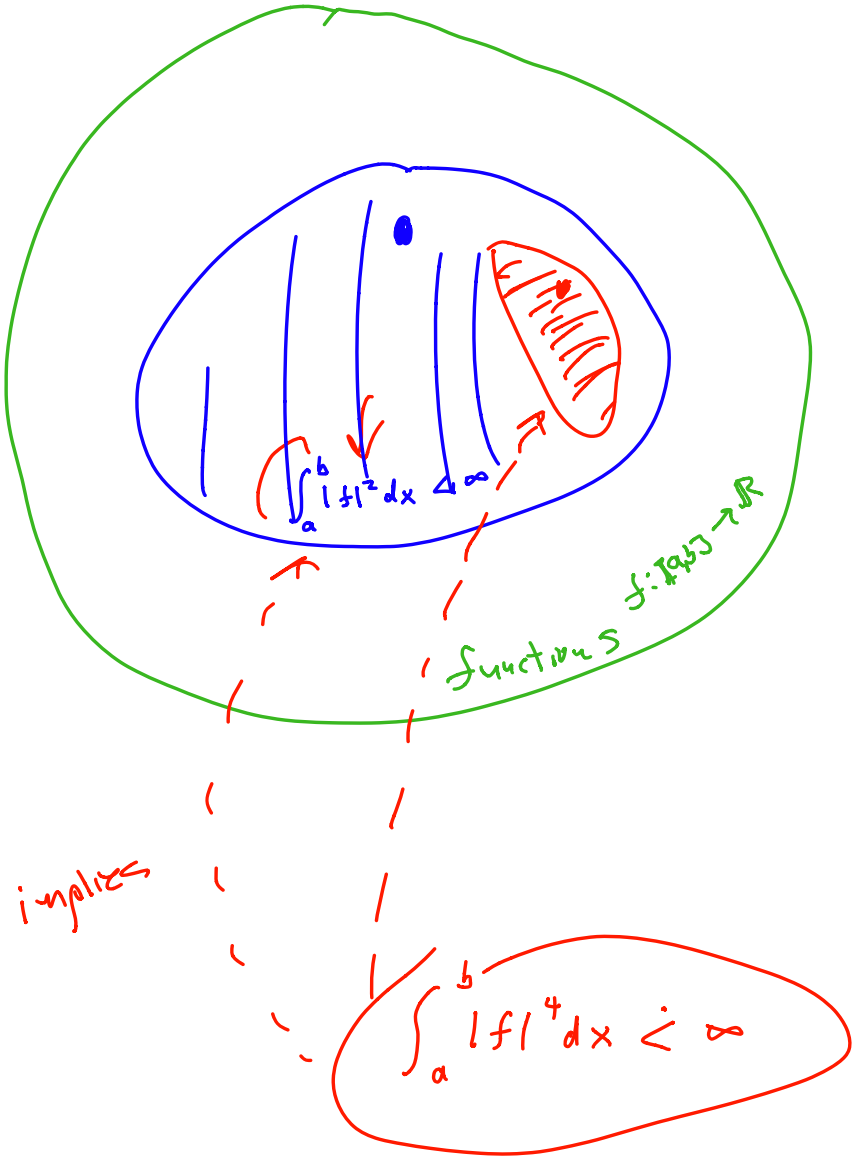
- HW: will include questions.
- HW: also, exercises (but few)



$$\int_a^b |f| + \int_a^b |\nabla f| + \int_a^b |\nabla^2 f| < \infty$$

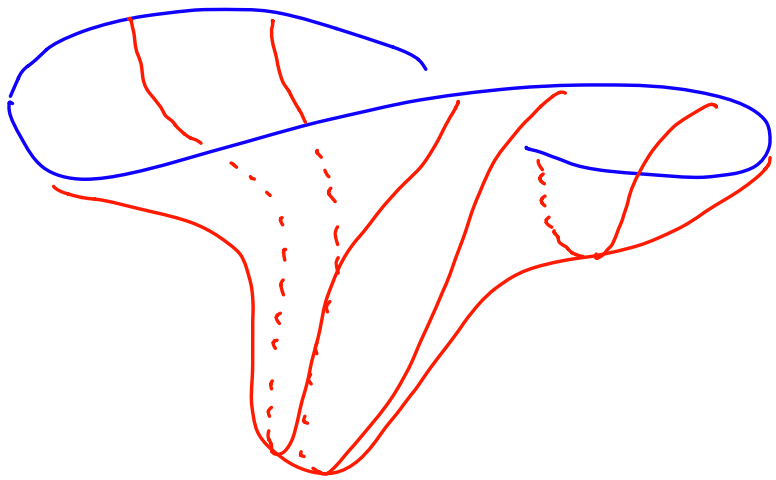
$$f^2 + g \cdot \nabla f + h^T \cdot \nabla^2 f \cdot h = K$$

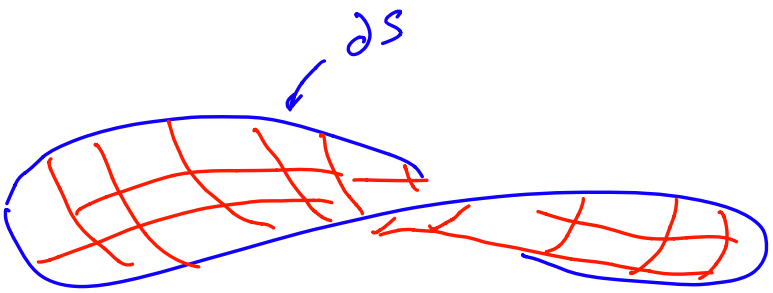
g: vector field

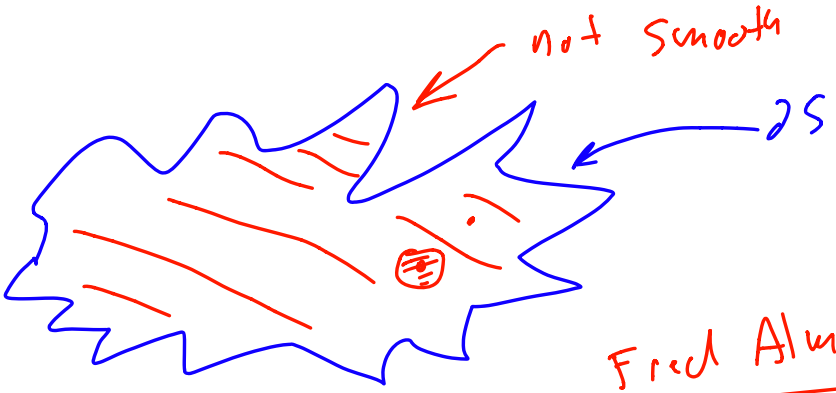
K: scalar

$f : \mathbb{R}^3 \to \mathbb{R}$

$g : \mathbb{R}^3 \to \mathbb{R}$







Fred Almgren

When $\dim \partial S \leq n-3$

where we are working in $\mathbb{R}^n$,

then the theorems about this

...

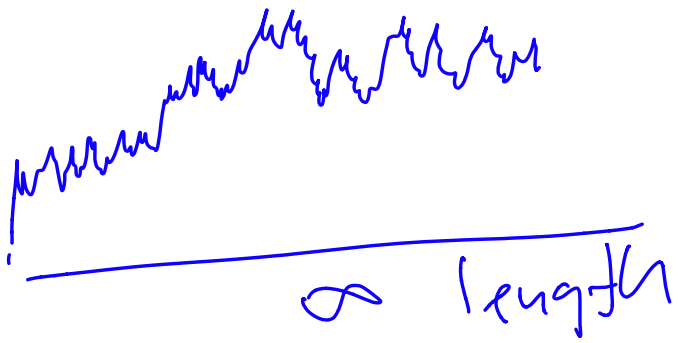
continuity $\supset$ differentiable

$\cup$

C^∞

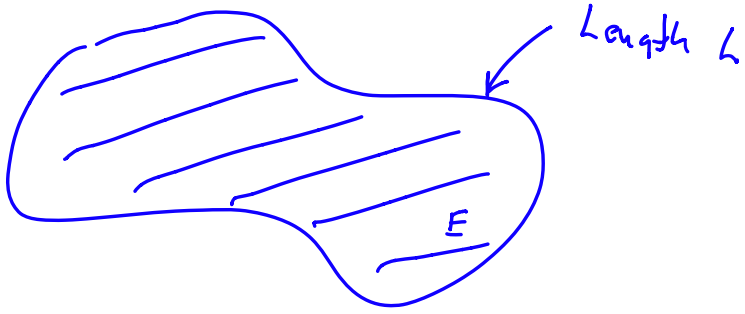
$\cup$

Analytic $\subset$

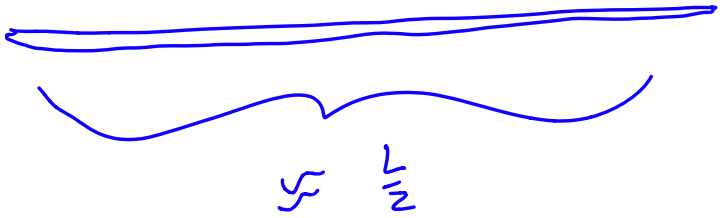


Isoperimetric Inequality

Sets in $\mathbb{R}^2$



What set with boundary length L has maximal area?



answer: disk of radius $r = \frac{L}{2\pi}$

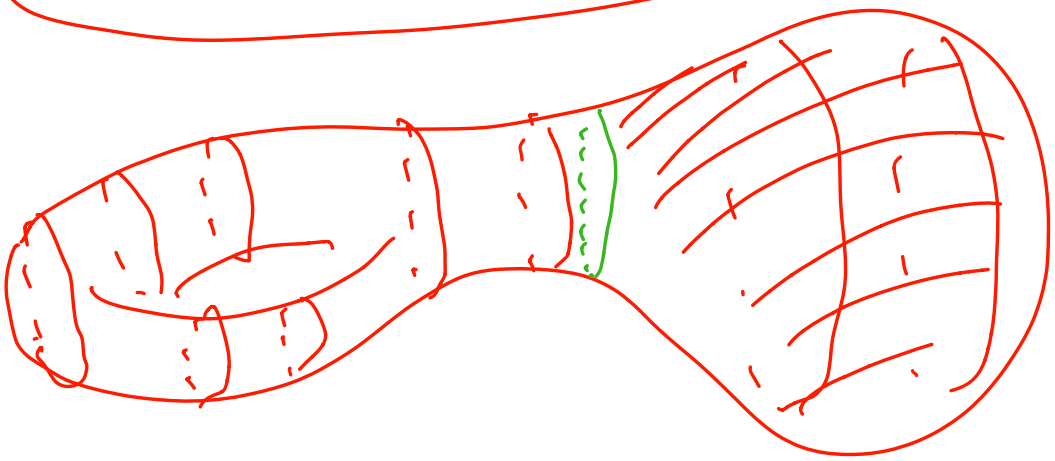
$$\frac{\pi \left(\frac{L}{2\pi}\right)^2}{A(E)} \geq 1$$

$$\frac{1}{4\pi} L^2 \geq A(E)$$

$$\frac{1}{4\pi} \left(\mathcal{H}^1(\partial E)\right)^2 \geq \mathcal{H}^2(E)$$

$$(H'(\partial E))^2 \geq 4\pi\, H^2(E)$$

$$L^2 \geq 4\pi A$$



Length squared ↘ $S^{\frac{3}{2}} \geq C V$ ↙ Length cubed

$$S \geq C V^{\frac{2}{3}}$$

$\frac{V_0}{N}$

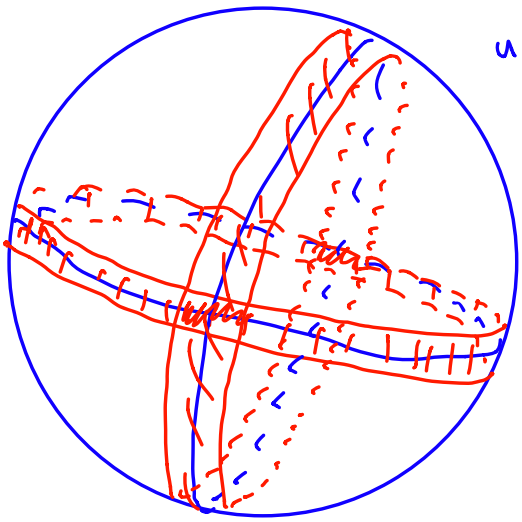
$$s_i \geq C\left(\frac{V_0}{N}\right)^{2/3}$$

$$\sum_{i=1}^{N} s_i \geq \sum_{i=1}^{N} C\left(\frac{V_0}{N}\right)^{2/3}$$

$$= C N \frac{V_0^{2/3}}{N^{2/3}}$$

$$= C N^{1/3} V_0^{2/3}$$

... Isoperimetric Inequality is a practical fact !!



Two vectors in high dimensional space chosen at random are close to orthogonal with high probability.

Look up the "one-pixel" camera.
