$x \cdot y \le |x||y|$ question

$\alpha \equiv |x| \neq 1$

$\beta \equiv |y| \neq 1$

$$v = \frac{x}{|x|} = \frac{1}{\alpha} x$$

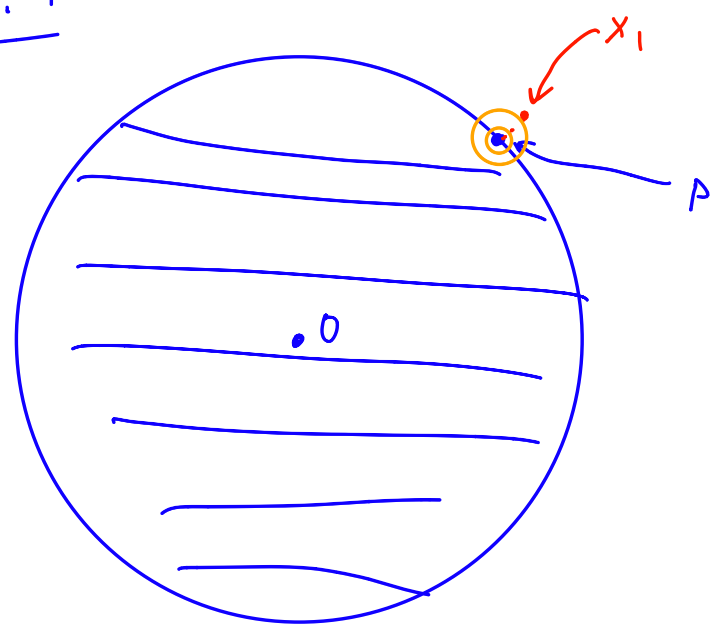
$$w = \frac{y}{|y|} = \frac{1}{\beta} y$$

$$v \cdot w \le |v| \cdot |w|$$

$$\frac{1}{\alpha} x \cdot \frac{1}{\beta} y \le \left| \frac{x}{|x|} \right| \left| \frac{y}{|y|} \right|$$

$$\frac{1}{\alpha\beta} (x \cdot y) \le \frac{1}{\alpha\beta} |x| |y|$$

4.7.4



$$|P - O| = 1$$

$$X_1 = 1.1\,P$$

$$X_2 = 1.01\,P$$

$$X_3 = 1.001\,P$$

$$X_4 = 1.0001\,P$$

$$\mathbb{Q} \cup \mathbb{Q}^c = \mathbb{R}^2$$

Question on Boundaries

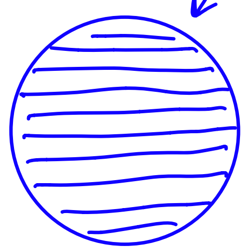
$E \qquad E^\circ, \partial E, (E^c)^\circ$

$$E^\circ \cup \partial E \cup (E^c)^\circ = \mathbb{R}^n$$

$\mathbb{Q}^n$

$\bar{B}(0,1)$

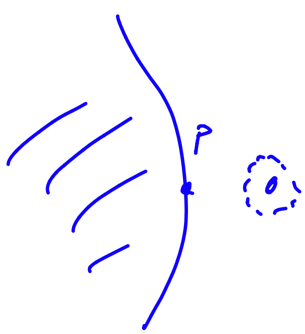
$= \{x \mid |x-0| \le 1\}$



$|x| > 1$

$|x < 1$

$|x| = 1$



p

$(1.00001)p$

$.999\,p$

$.4 \quad 1$

① $\bigcup_{\alpha \in A} O_\alpha$ is also open

② D closed $\Rightarrow$ D^c open
D open $\Rightarrow$ D^c closed

arbitrary intersections of closed sets

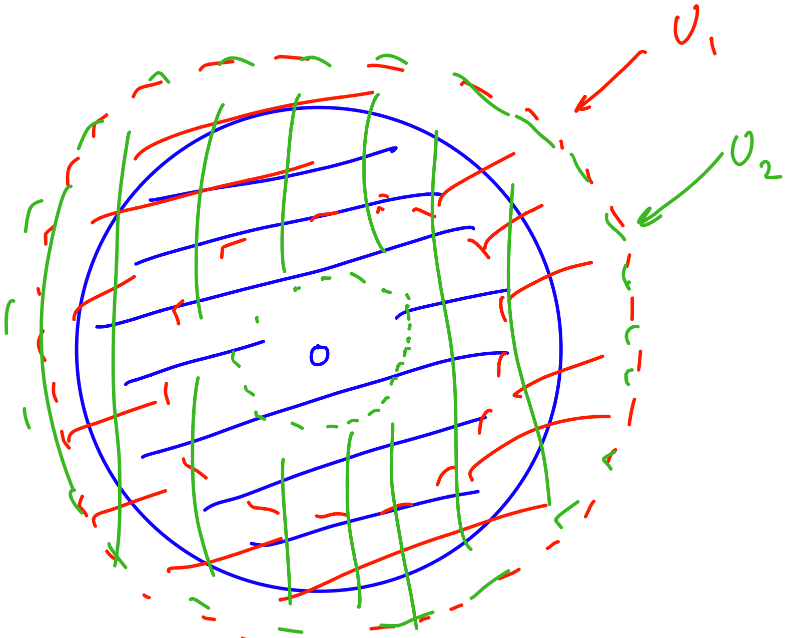
$\{C_\alpha\}_{\alpha \in A}$

$\bigcap_{\alpha \in A} C_\alpha$ also closed ?

$$\bigcap_{\alpha \in A} C_\alpha = \left(\bigcup_{\alpha \in A} C_\alpha^c \right)^c$$

$$\left[\left(\bigcap_{\alpha \in A} C_\alpha \right)^c = \bigcup_{\alpha \in A} C_\alpha^c \right]$$

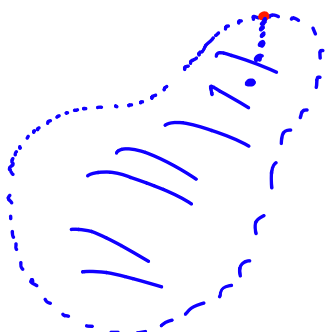
4.7.7



$$U_i = \{x \mid \tfrac{1}{2^i} < |x| < 1.1\}$$

$$\mathrm{cl}(E) = \bigcap_{\substack{D \text{ closed} \\ E \subset D}} D$$

$$\mathrm{cl}(E) = \{ x \mid \exists \{y_i\}_{i=1}^{\infty} \subset E \quad y_i \to x \}$$



$$\mathrm{cl}(E) = E \cup \partial E$$

$$E \text{ closed} \iff E = \mathrm{cl}(E)$$

4.7.3

$$M = (|x-y|, \mathbb{R}^2)$$

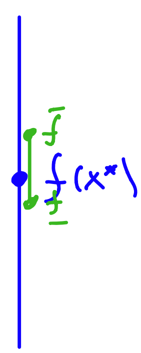
$$\Downarrow$$

usual Euclidean space/plane

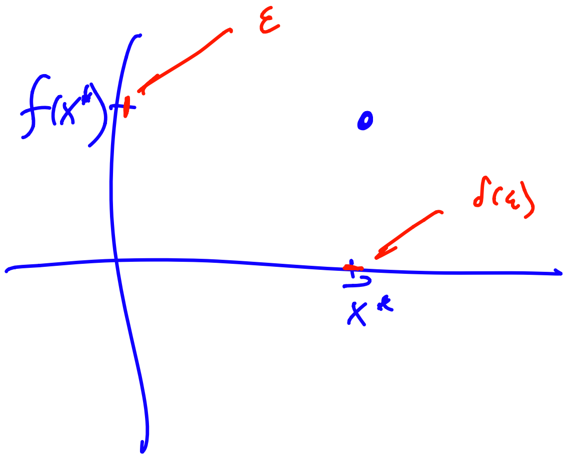
4.6.1

$$\underbrace{\liminf_{x \to x^*} f(x)}_{\underline{f}} \le f(x^*) \le \underbrace{\limsup_{x \to x^*} f(x)}_{\overline{f}}$$

(1)

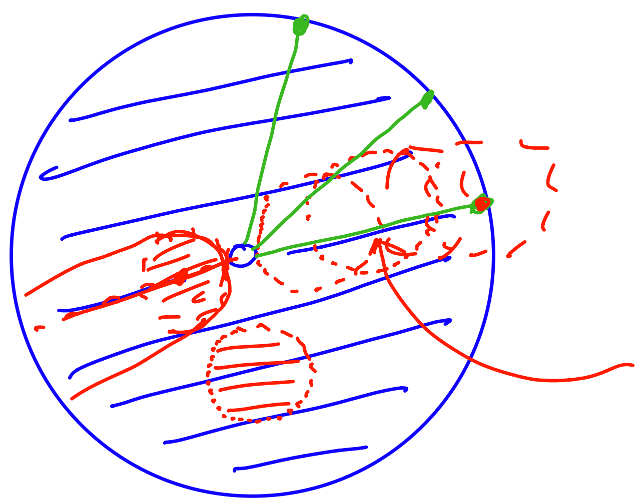


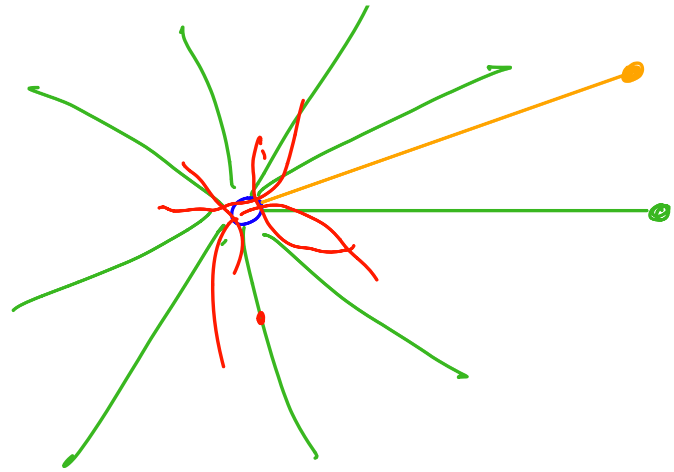
$$\overline{f} - \underline{f} = \eta > 0$$



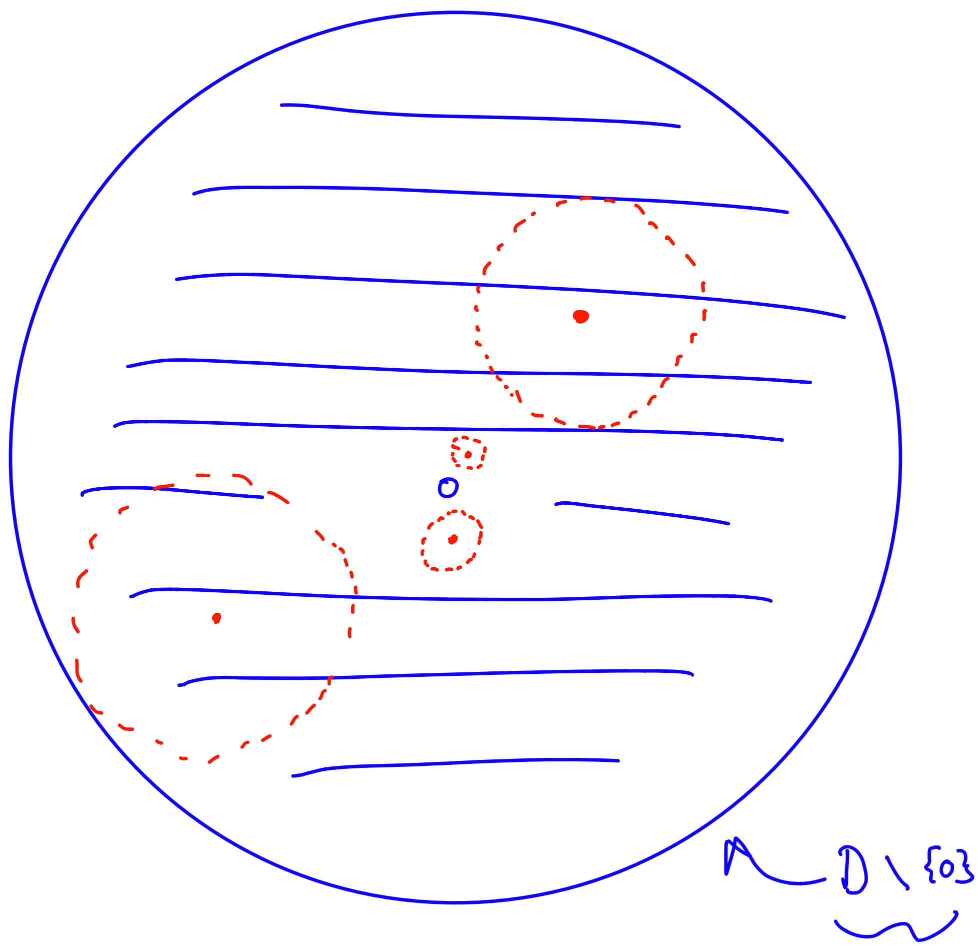
$\varepsilon < \frac{M}{2}$

$\Rightarrow \in$





Katrina's version of 4.7.7



$\forall x \in D \setminus \{0\}$

$B(x, \frac{|x|}{2})$

$$B(0, \tfrac{|x|}{2}) \cap B(x, \tfrac{|x|}{2}) = \phi$$

Claim $\bigcup_{i=1}^{N} B(x_i, \frac{|x_i|}{2}) \supset D \setminus \{0\}$

<u>But</u> $\exists$ a closest point x_K to origin

i.e. $x_K \leq x_i \quad \forall i$

$\Rightarrow \bigcap_{i=1}^{N} B(0, \frac{|x_i|}{2})$ is <u>not</u> covered

$\Rightarrow B(0, \frac{|x_K|}{2}) \subset \bigcap_{i=1}^{N} B(0, \frac{|x_i|}{2})$

is <u>not</u> covered

$\Rightarrow$ claim is false