$K \to f(K)$

$*$ f is continuous , $\mathbb{R}^n \to \mathbb{R}$

and K is compact, then

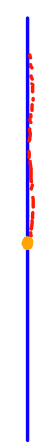
$\exists\ x_m , x_M \ni f(x_m) = \inf_{x \in K} f(x)$

infimum = greatest lower bound

$f(x_M) = \sup_{x \in K} f(x)$

$**$ $\boxed{\text{f continuous} \Rightarrow \text{f maps compact sets to compact sets.}}$

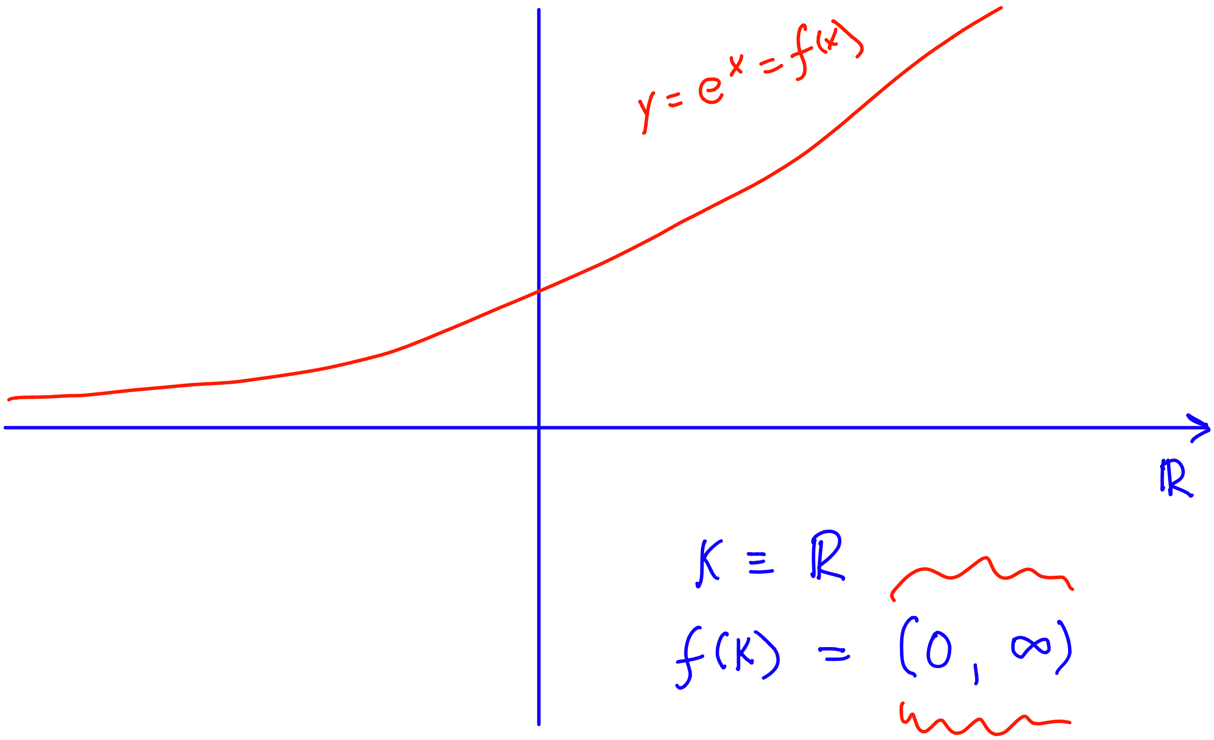
$* \ \partial \ ** \Rightarrow f(K)$ is compact



f(K) is closed

$\Rightarrow$ • is in closure of

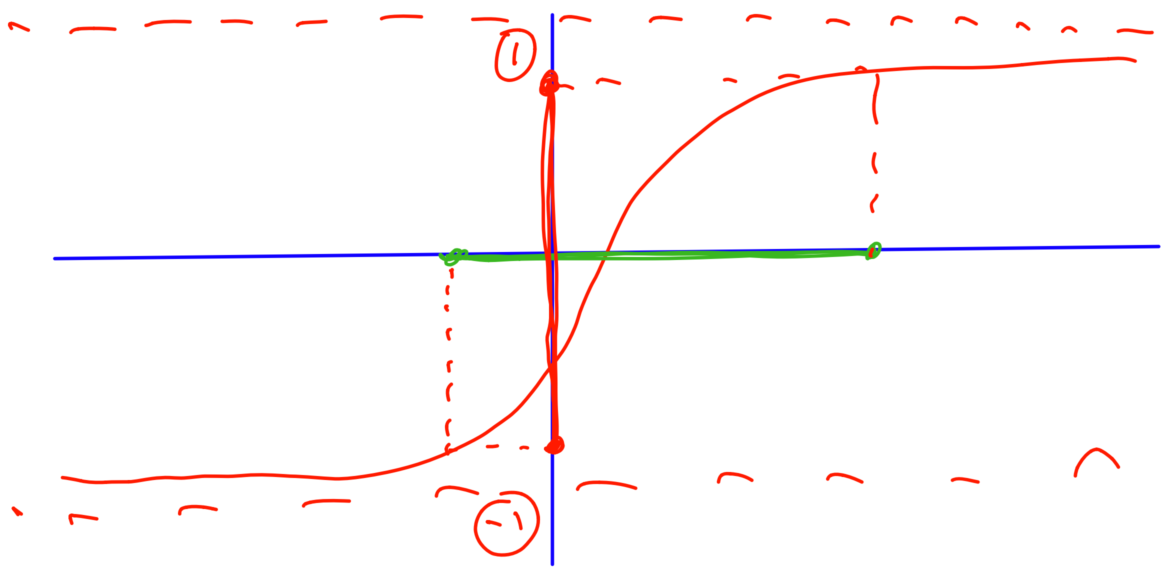
$\Rightarrow$ • $\in f(K)$

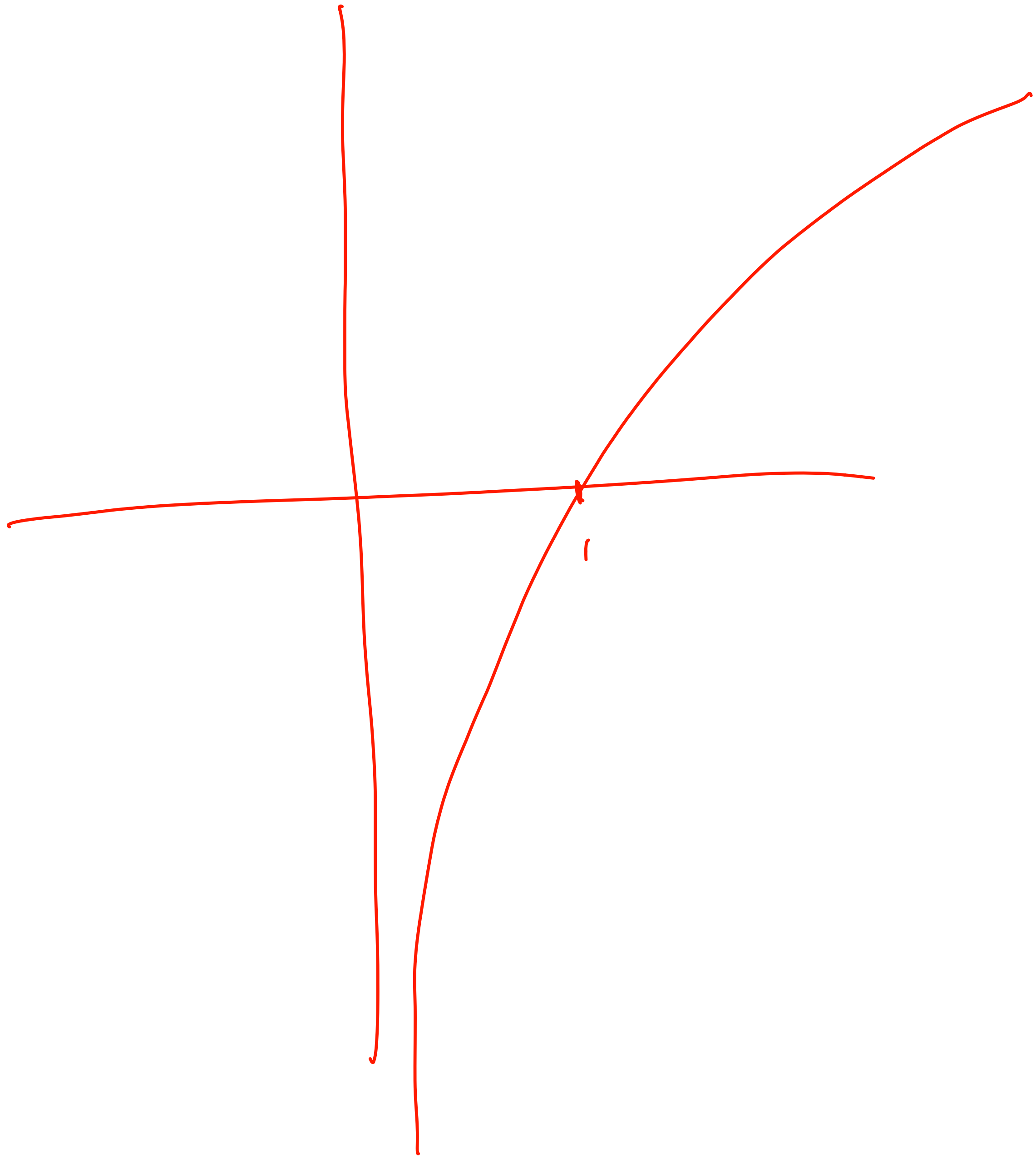



$K = \mathbb{R}$

$f(K) = (0, \infty)$

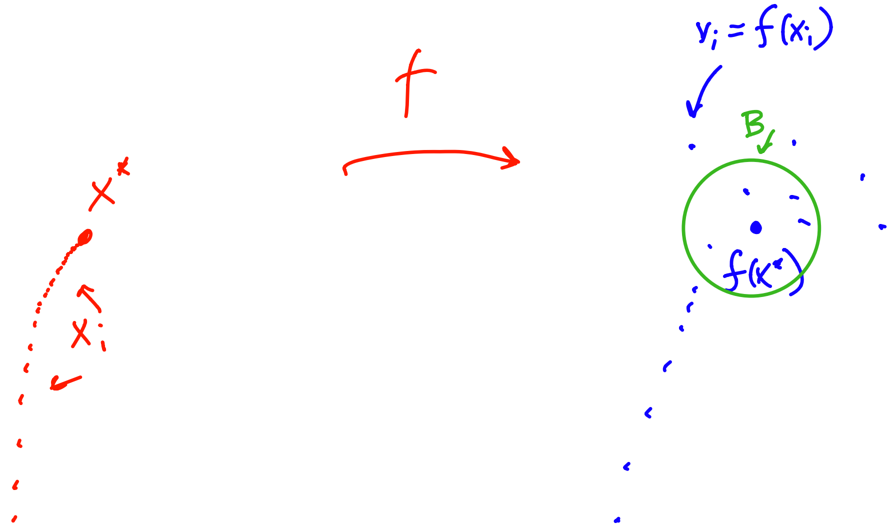
$0 \overset{?}{\in} (0, \infty)$



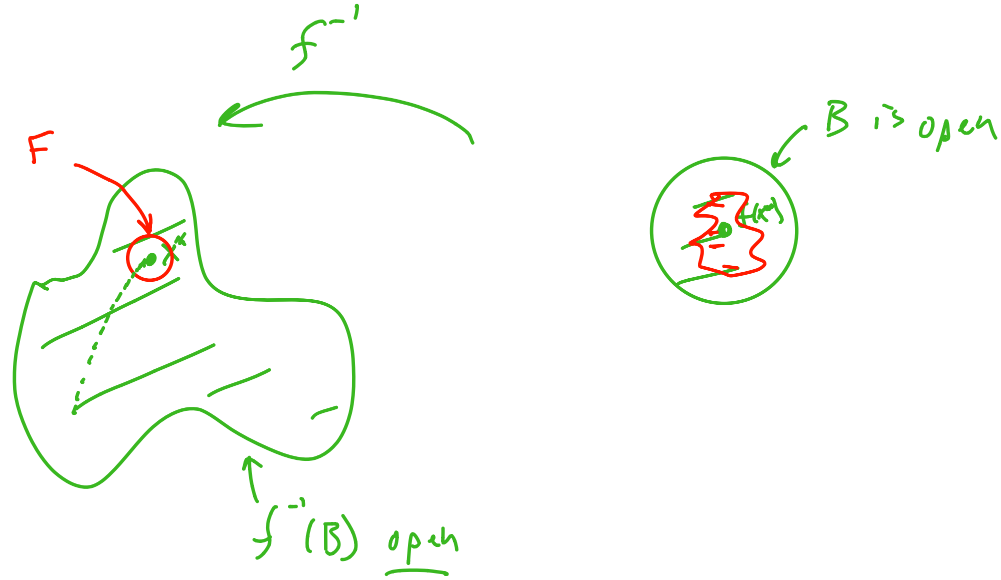


① $f^{-1}(E)$ is open when $E$ open

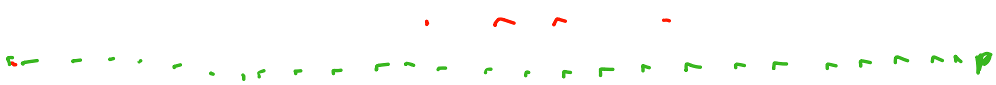
② $\exists \underbrace{x_i \to x^* \text{ s.t. } f(x_i) \not\to f(x^*)}_{\text{make this concrete}}$




always red dots to right




$F \subset f^{-1}(B)$



eventually no red dots to right.