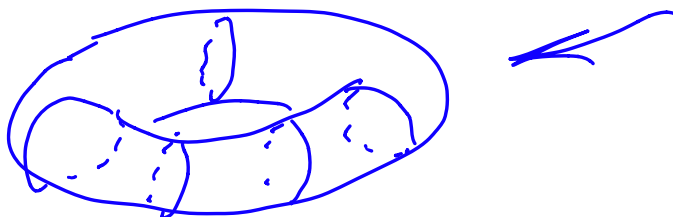


$\mathbb{R}^n$  is a metric space  
(implicitly choosing the Euclidean norm)

2-Torus

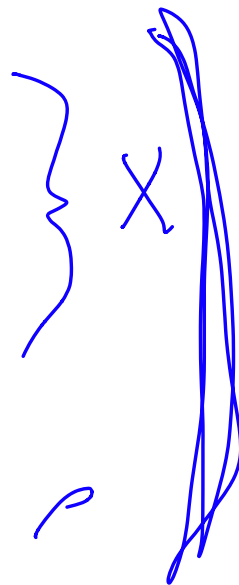


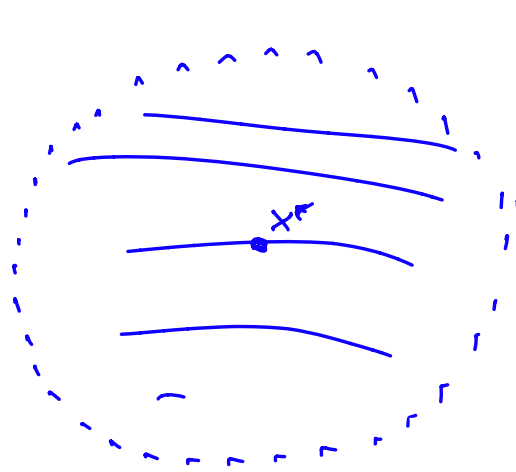
function space  $C([0, 1], \mathbb{R})$

space of continuous functions

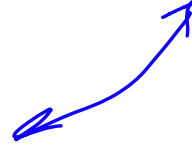
$$f: [0, 1] \rightarrow \mathbb{R}$$

$$\rho(f, g) \equiv \max_{x \in [0, 1]} |f(x) - g(x)|$$





$$\{x \mid p(x, x) < 3\}$$



$$(x^2 - 1) = (x+1)(x-1)$$



calculus or polynomials

cryptograph y

$$25 = 5^2$$

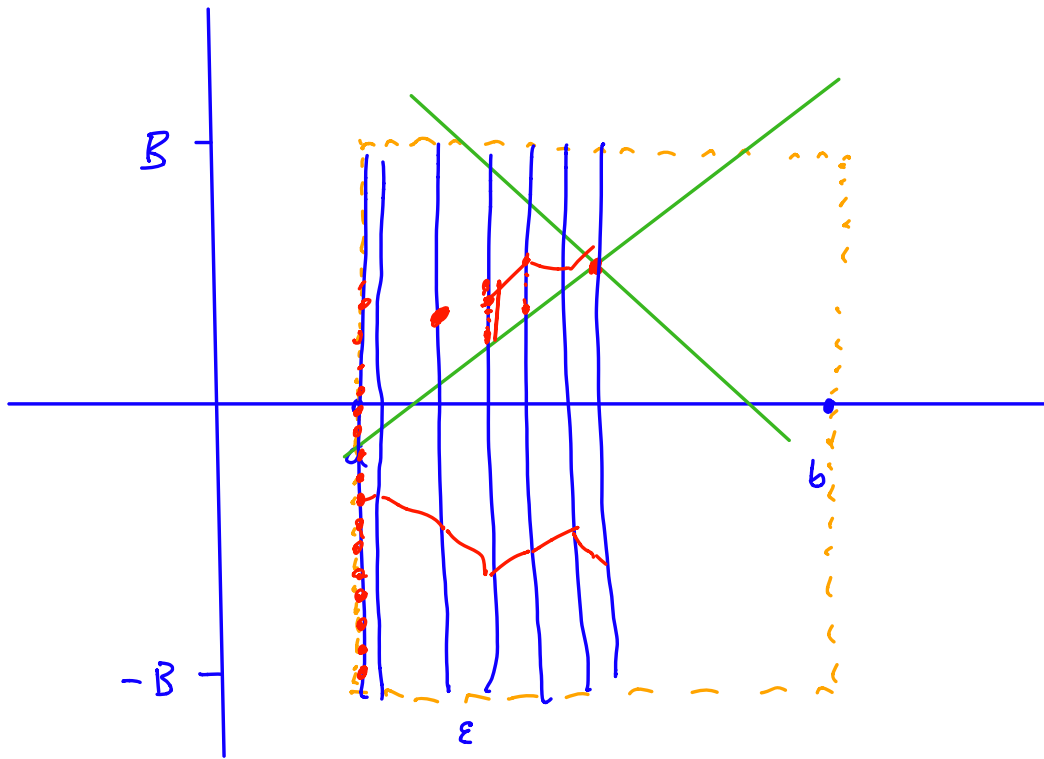
"Range" David Epstein

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$$\begin{array}{ccc}
 a & \leq & \xi \\
 & \approx & \xi \\
 & \leq & \xi
 \end{array}$$

notes on  
EX 4.7.15

$Lip([a,b], K, B)$



$$\frac{\sum B}{\epsilon} = \frac{2K\epsilon}{\epsilon} = 2K$$

$$\left(\frac{B}{\epsilon}\right) (2K) (2K) (2K) \dots (2K)$$

1      2      3       $\frac{b-9}{\epsilon}$

$$\left(\frac{B}{\epsilon}\right) (2K)^{\left(\frac{b-9}{\epsilon}\right)}$$

$$\left(\frac{B}{\epsilon}\right) (2K)^{\left\lceil \frac{b-9}{\epsilon} \right\rceil}$$

IF Lipschitz reminder

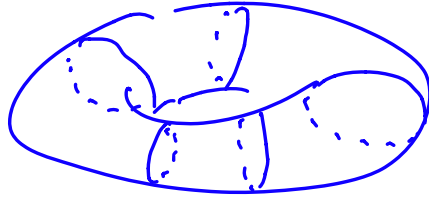
$$|f(x) - f(y)| < K |x - y|$$

for  $K < \infty$   $\forall x, y$  in domain

$f$  is said to be Lipschitz continuous

# uses of Lipschitz functions/mappings

manifold →



←  
Rectifiable sets  
generalize and  
they are defined  
using Lipschitz  
mappings

$$\dot{x} = f(x) \text{ autonomous}$$

proving  
existence and  
uniqueness for  
differential  
Equations

$$\dot{x} = f(x, t) \text{ non-autonomous}$$

→ Picard-Lindelöf

uses Lipschitz  
condition on the  
R.H.S. (right hand  
side)  $f$ .

→ Peano

← another existence theorem  
that does not use  
Lipschitz ... but you  
do not get uniqueness!