$\rightarrow$ $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3 \leftarrow$ vector spaces

- how can metric spaces differ from these?
- linear vs. nonlinear ... and why this question assumes your metric space is a vector space.

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nice vector space. $\rightarrow$ V = some vector space (assume it is normed)

M = some metric space

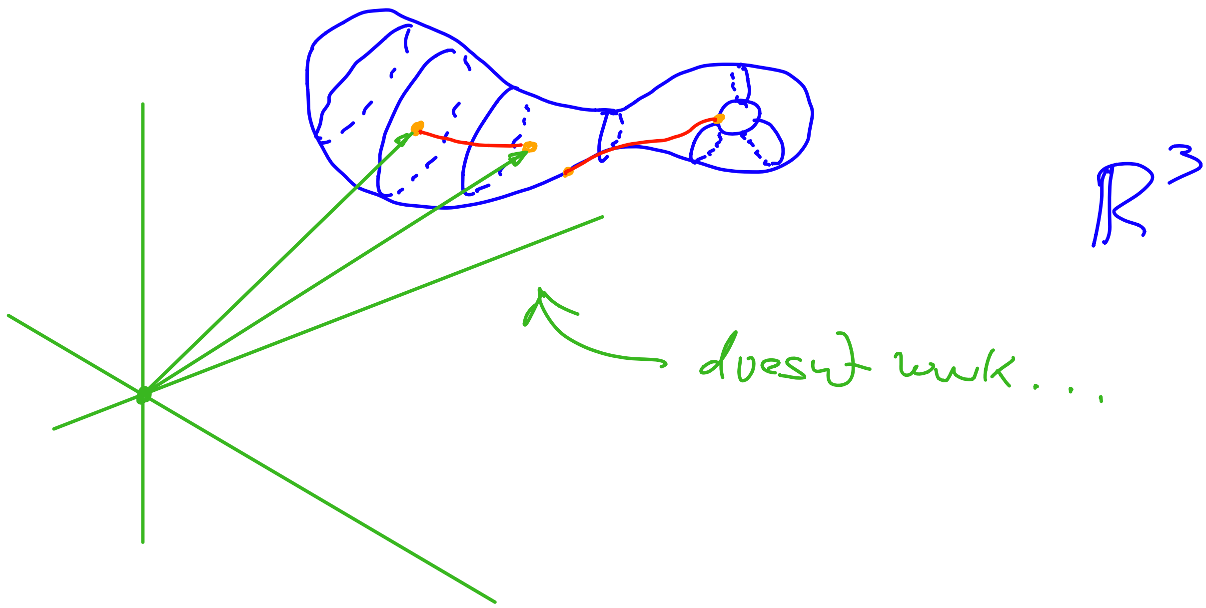
$\|x\| = \sqrt{\langle x, x \rangle}$

Differences:

- $x, y \in V \quad \alpha, \beta \in \mathbb{R}$
  then $\alpha x + \beta y \in V$

  vs.

  this does not make sense in a general metric space.



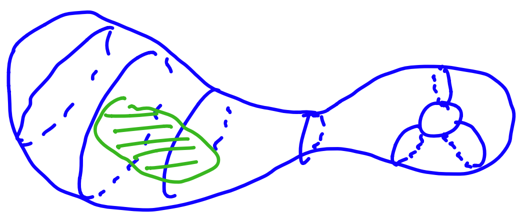
• if $f: M \to \mathbb{R}$ can I compute $\frac{df}{dx}$

$$\frac{f(x+h) - f(x)}{|h|}$$

can't do derivatives!!

things that are the "same":

- I can integrate:



Hausdorff measure μ

$$\int f \, d\mu$$

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

f satisfies $\Leftrightarrow$ f linear

$$f : \mathbb{R}^4 \to \mathbb{R}$$

$$\mathbb{R} \to \mathbb{R}$$

$$\mathbb{R}^4 \; (x_1, x_2, x_3, x_4) \longrightarrow \mathbb{R} \; (y)$$

$$\underbrace{x_1 (1,0,0,0)}_{e_1} + \underbrace{x_2 (0,1,0,0)}_{e_2} + x_3 \overbrace{(0,0,1,0)}^{e_3} + x_4 \underbrace{(0,0,0,1)}_{e_4}$$

$$f(x_1 e_1 + x_2 e_2 + x_3 e_3 + x_4 e_4)$$

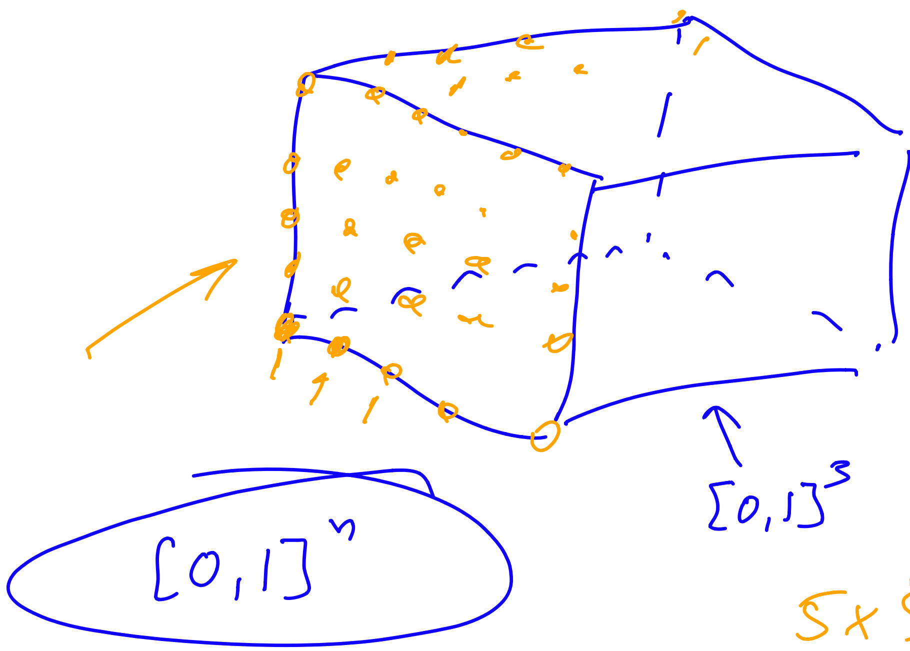
$$\parallel$$

$$x_1 f(e_1) + x_2 f(e_2) + x_3 f(e_3) + x_4 f(e_4)$$

$$1 \; [\underset{f(e_1)}{\alpha_1}, \underset{f(e_2)}{\alpha_2}, \underset{f(e_3)}{\alpha_3}, \underset{f(e_4)}{\alpha_4}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$f : \mathbb{R}^{23} \longrightarrow \mathbb{R}^{15}$$

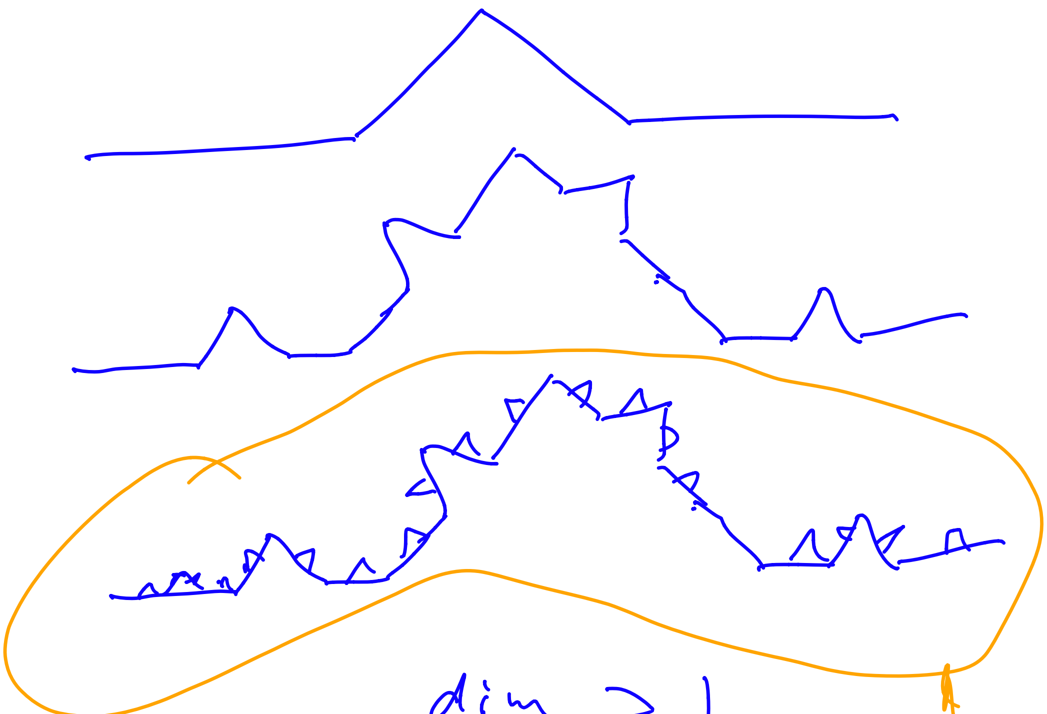
$15 \cdot 23 = 345$ numbers



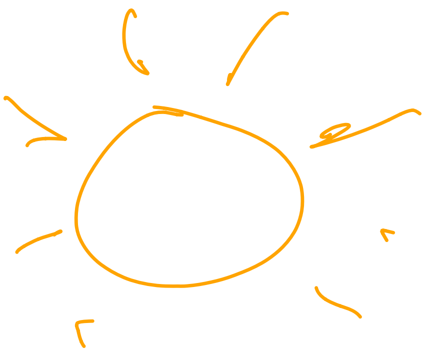
$5 \times 5 \times 5 = 125$

$\mathbb{R}^{23} \to \mathbb{R}$ 23 numbers

$5^{23} \to$ Big number



dim > 1



dla

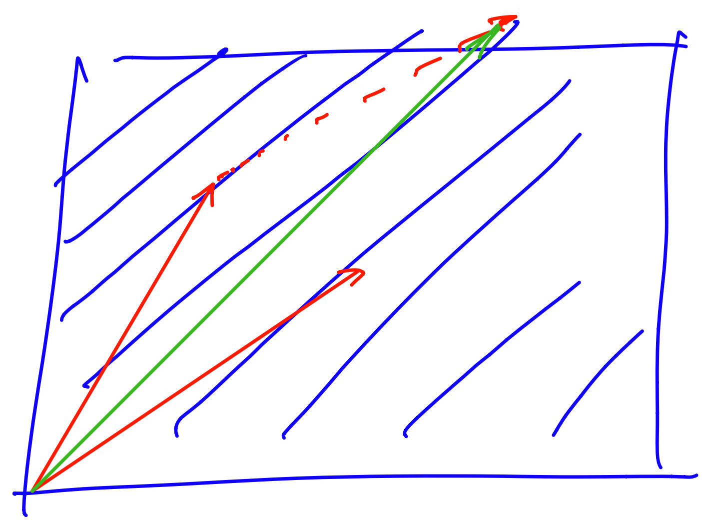
diffusion limited aggregation

Mandelbrot Autobiography

Questions ?

vector space that is not a metric space?

$\mathbb{R}^2$



$\rightarrow X = \underbrace{\{ f : \mathbb{R} \to \mathbb{R} \}} \leftarrow$

(a) clearly multiplication by scalars is defined

(b) clearly addition is defined

(c) <u>but</u> ... so problems

$\infty \neq \rho(x,y) \in \mathbb{R}^+$

usually

$X = \{ f : \mathbb{R} \to \mathbb{R}$ <u>and</u> some "regularity" condition $\}$

$$\boxed{\rho(\alpha x, \alpha y) = \cancel{\alpha} \rho(x,y)}$$