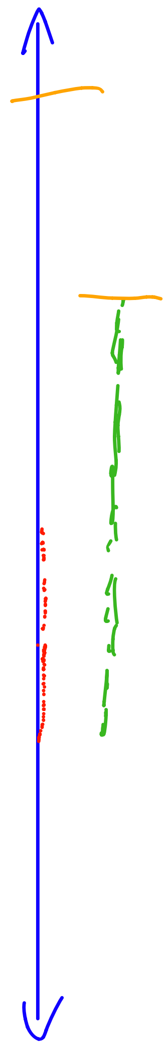


$\{x_i\}_{i=1}^{\infty} \subset \mathbb{R}$

$$\boxed{\begin{array}{c} x_i \le x_j \\ i < j \end{array}}$$

show this implies that

$\exists \; x^* \ni x_i \to x^*$

proof: (1) $x^* \equiv \sup \{x_i\}_{i=1}^{\infty} \Leftarrow X$

(2)

(a) define

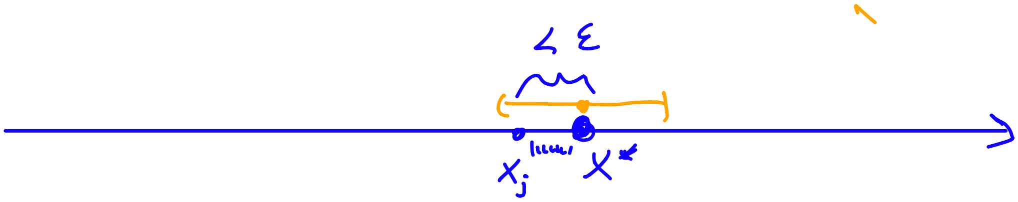
$\sup E \equiv$ smallest upper bound

(b) real numbers have the property that sup's always exist.

(3) define $x^* \equiv \sup X$, $\varepsilon > 0$

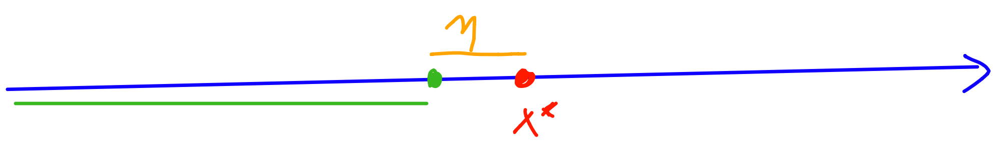
claim: $\exists\, j \ni x^* - x_j < \varepsilon$

$x_{j+1} \quad x_{j+2} \quad x_{j+3} \quad \cdots$



Suppose not: then for some $\eta > 0$

$$x^* - x_k \geq \eta \quad \forall\, k$$



$$X \left\{ \begin{array}{l} x_1 = 1 - \frac{1}{1} \\ x_2 = 1 - \frac{1}{2} \\ x_3 = 1 - \frac{1}{3} \\ x_4 = 1 - \frac{1}{4} \end{array} \right\} \Rightarrow \sup X \notin X$$

$x_1 = 1 - \frac{1}{1}$

$x_2 = 2 - \frac{1}{2}$

$x_3 = 1 - \frac{1}{3}$

$x_4 = 2 - \frac{1}{4}$

$x_5 = 1 - \frac{1}{5}$