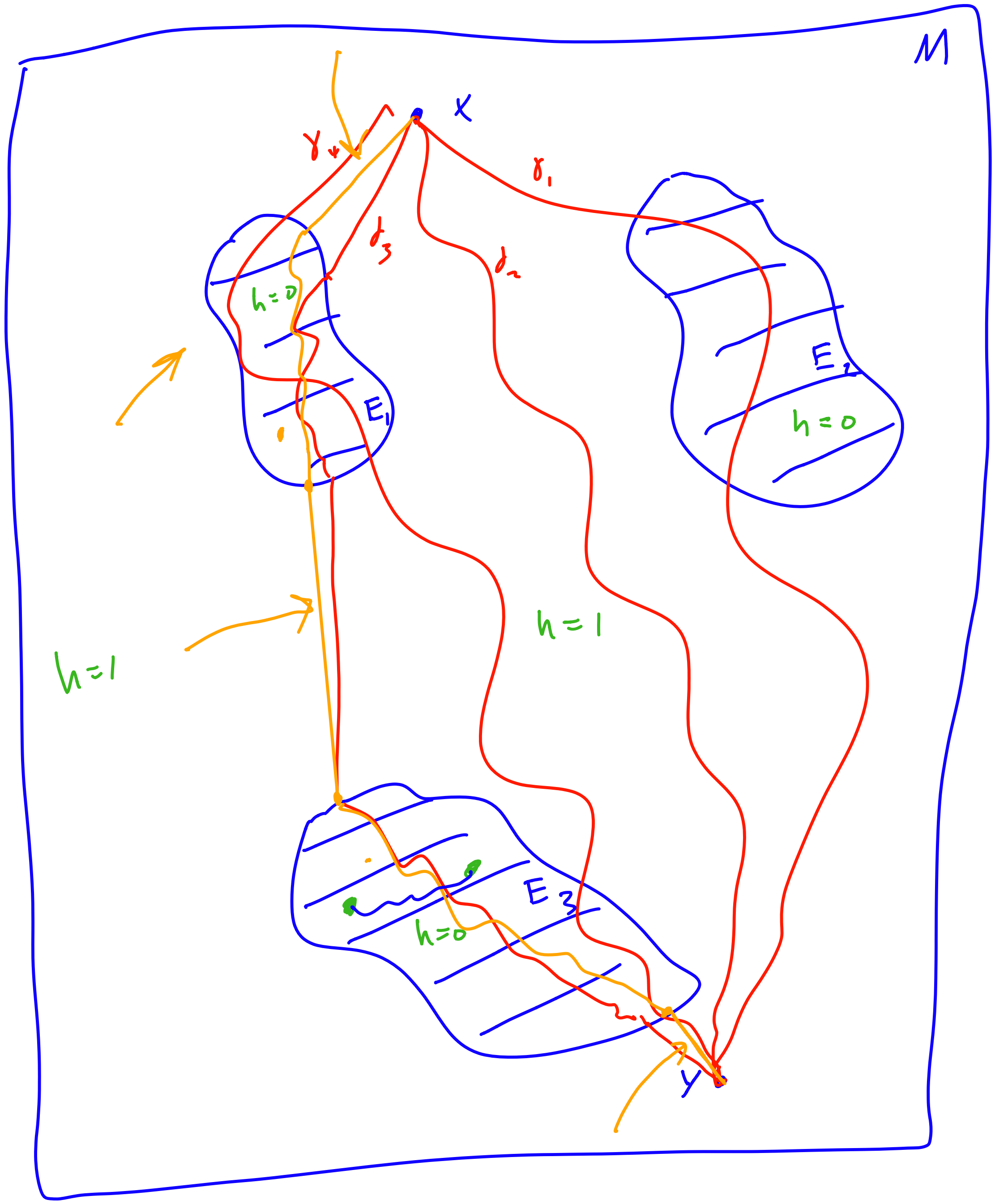


$$L(\gamma_1) = \int_0^1 h(\gamma_1(t))\, \dot{\gamma}_1(t)\, dt$$

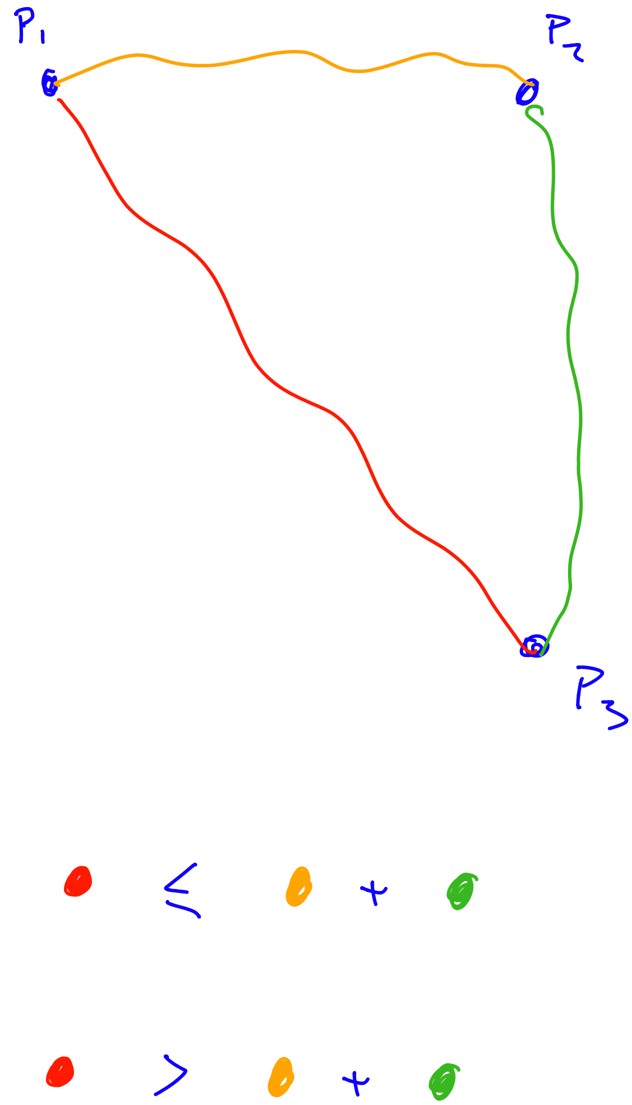
$\gamma_1 : [0,1] \to M \quad \gamma_1(0) = x \quad \gamma_1(1) = y$

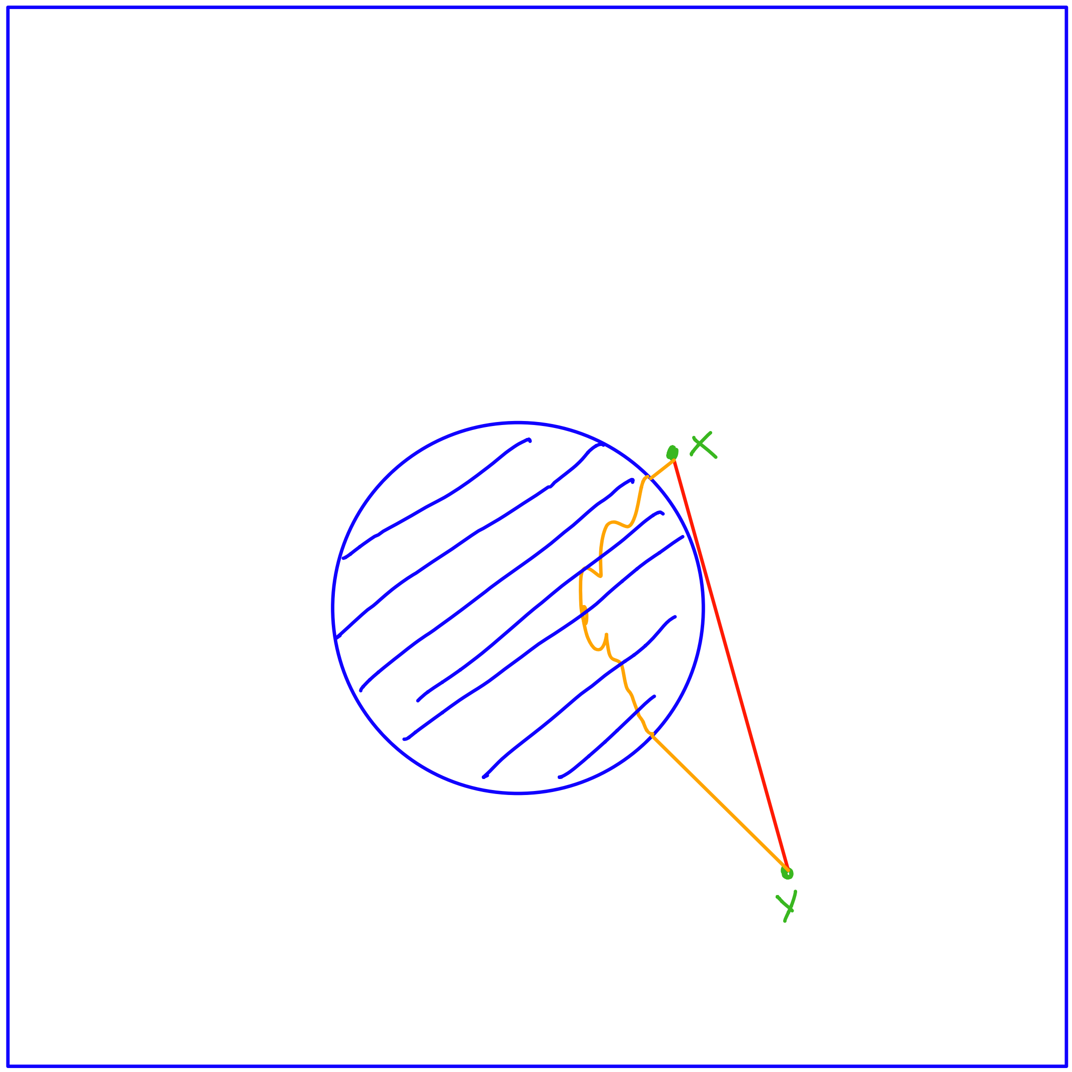
$$h(\gamma_1(t))\,\dot{\gamma}_1(t)\,dt = 1 \cdot \underline{\dot{\gamma}_1(t)\,dt}$$

$$\int_0^1 \dot{\gamma}(t)\,dt = \text{length of image of } \gamma$$

$$\boxed{\rho(x,y) \equiv \inf_{\substack{\gamma \\ \gamma(0)=x \\ \gamma(1)=y}} L(\gamma) = \inf_{\substack{\gamma \\ \gamma(0)=x \\ \gamma(1)=y}} \int_0^1 h(\gamma(t))\,\dot{\gamma}(t)\,dt}$$

$$\underline{M = \mathbb{R}^2 \setminus E}$$


P_1
P_2
P_3
≤ +
> +


X
Y

$M = (\mathbb{R}^2, \rho)$

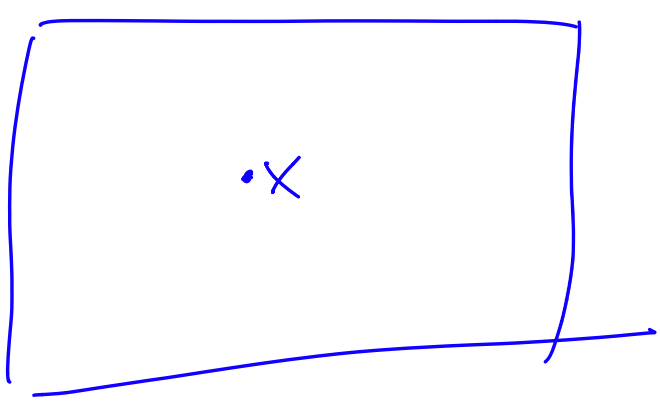
$\rho(x,y) = 1 \quad x \neq y$

$\rho(x,y) = 0 \quad x = y$

$\rho(x,z) \le \rho(x,y) + \rho(y,z)$

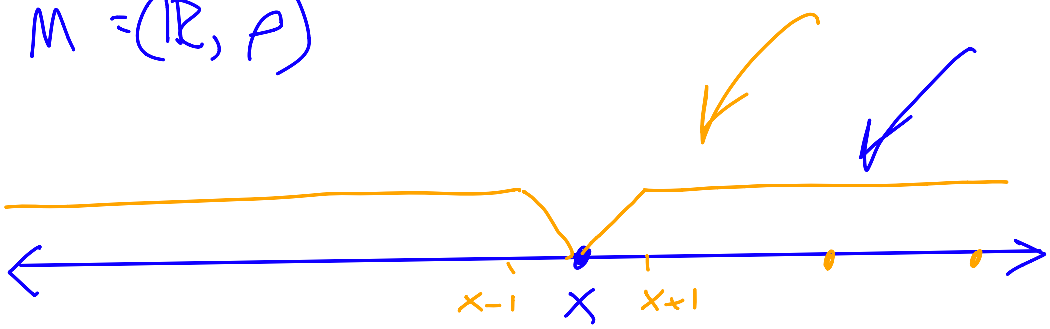
$B(x, \tfrac{1}{2}) = \{x\}$

$\Rightarrow$ <u>all</u> sets are open

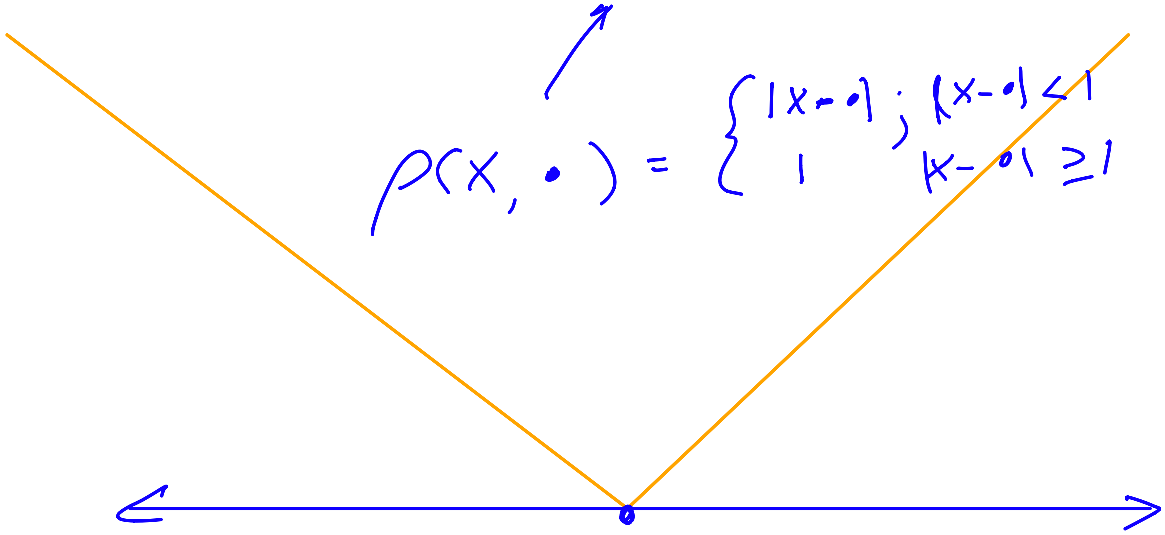


<u>$\mathbb{R}^2 \setminus \{x\}$</u> <u>not</u> dense in $\mathbb{R}^2$

$M = (\mathbb{R}, \rho)$

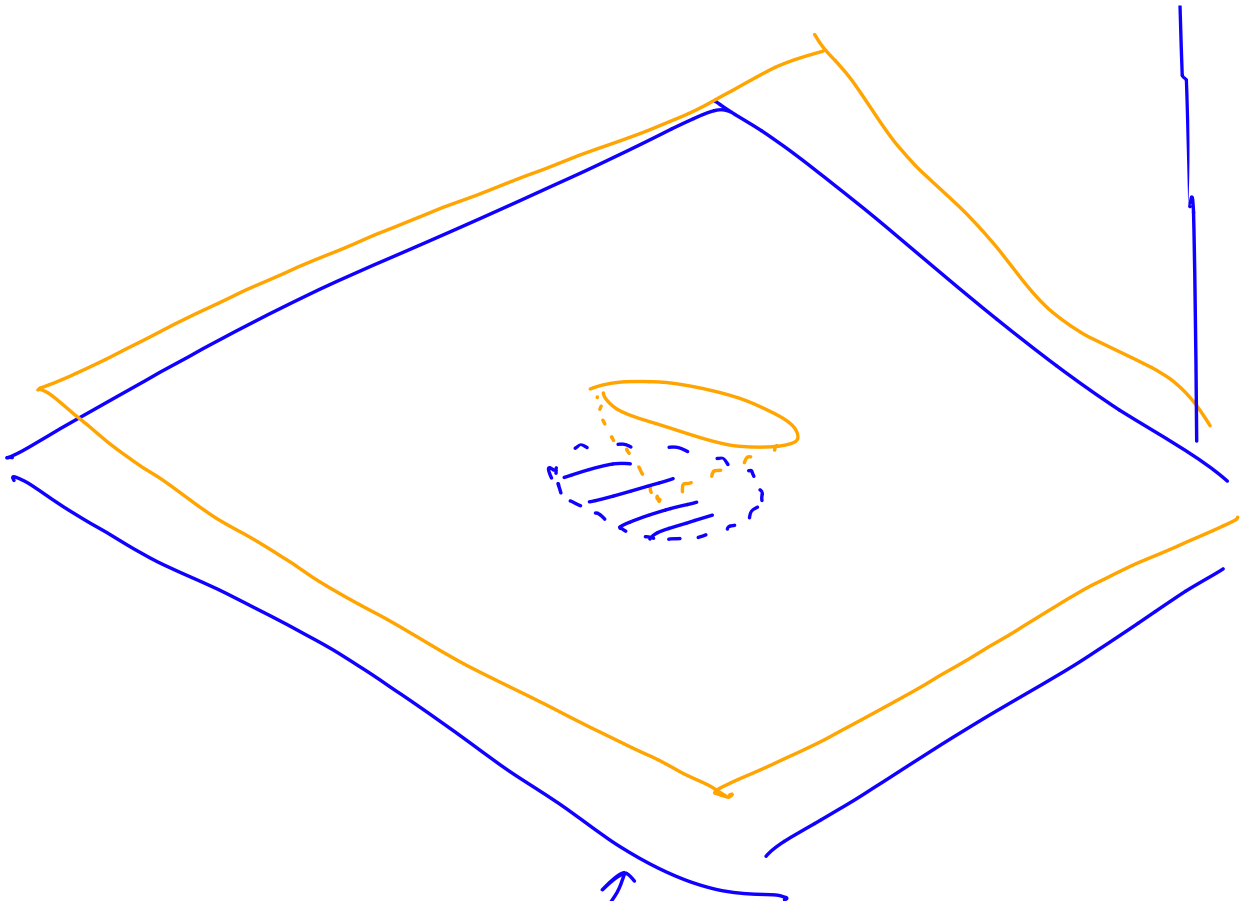


$$\rho(x, 0) = \begin{cases} |x-0| ; & |x-0| < 1 \\ 1 & |x-0| \geq 1 \end{cases}$$



$\rho(x, 0)$ for the usual metric on $\mathbb{R}$

$$= |x - 0|$$



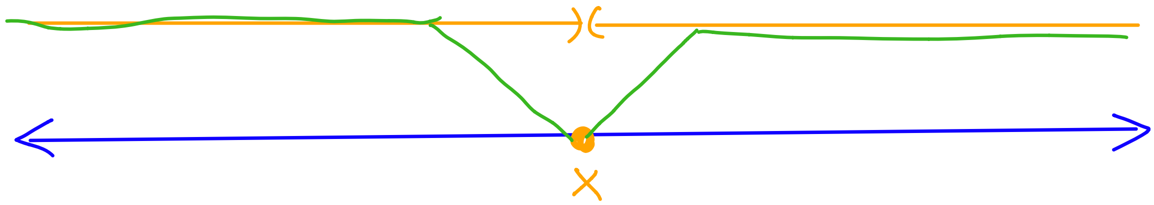
$\mathbb{R}^2$

$f(y) \equiv |x - y|$

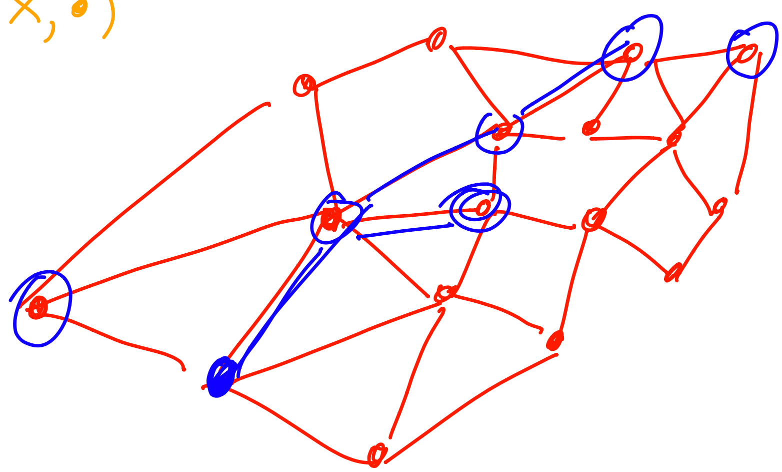
x fixed

y variable

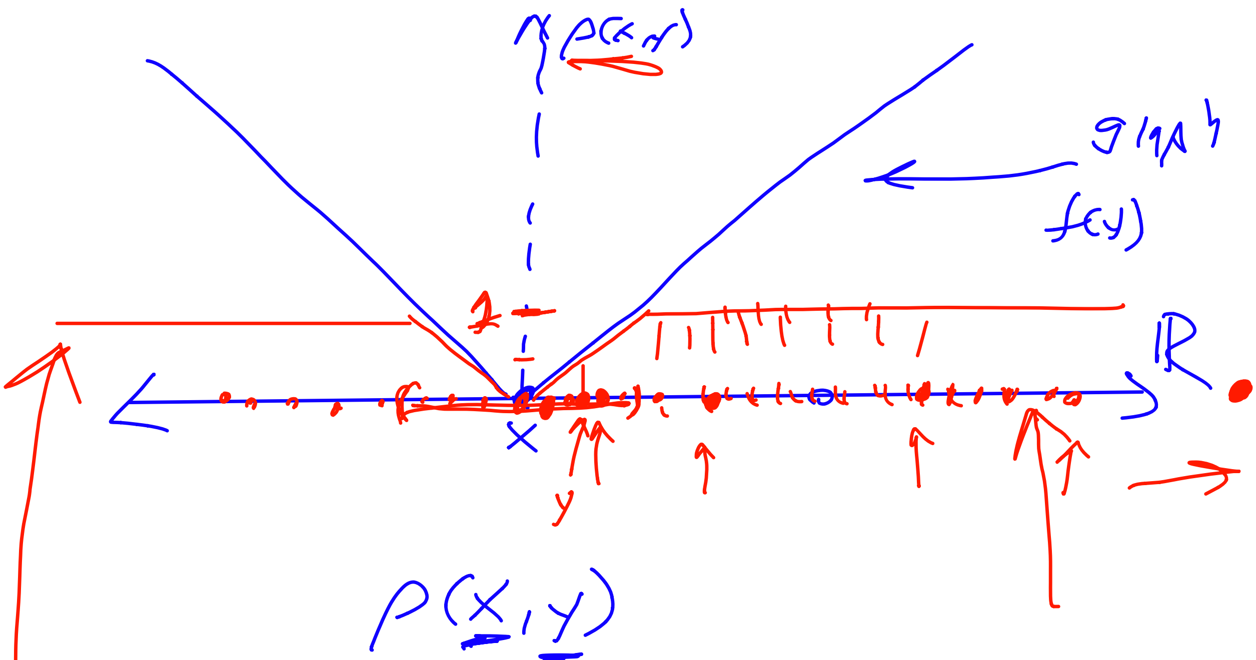
"draw this" until it is clearly a cone (graph) discrete metric in $\mathbb{R}$



$\rho(x,\cdot)$



Paul Erdos $\leftarrow$ 1600



$$\rho(x, y)$$

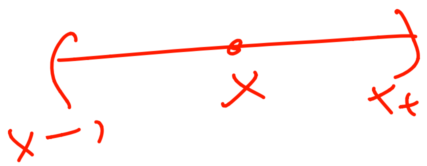
$$f(y) = |x - y|$$

$$f(y) = \begin{cases} |x-y| & \text{when } |x-y| \le 1 \\ 1 & \text{when } |x-y| > 1 \end{cases}$$

$B(x, 2) \leftarrow \mathbb{R}$

$y \in \mathbb{R} \Rightarrow \rho(x, y) < 2$

$B(x,1)$



$B(x, 1+\varepsilon) = \mathbb{R}$

$\{ y \mid \rho(x,y) < 1 \}$