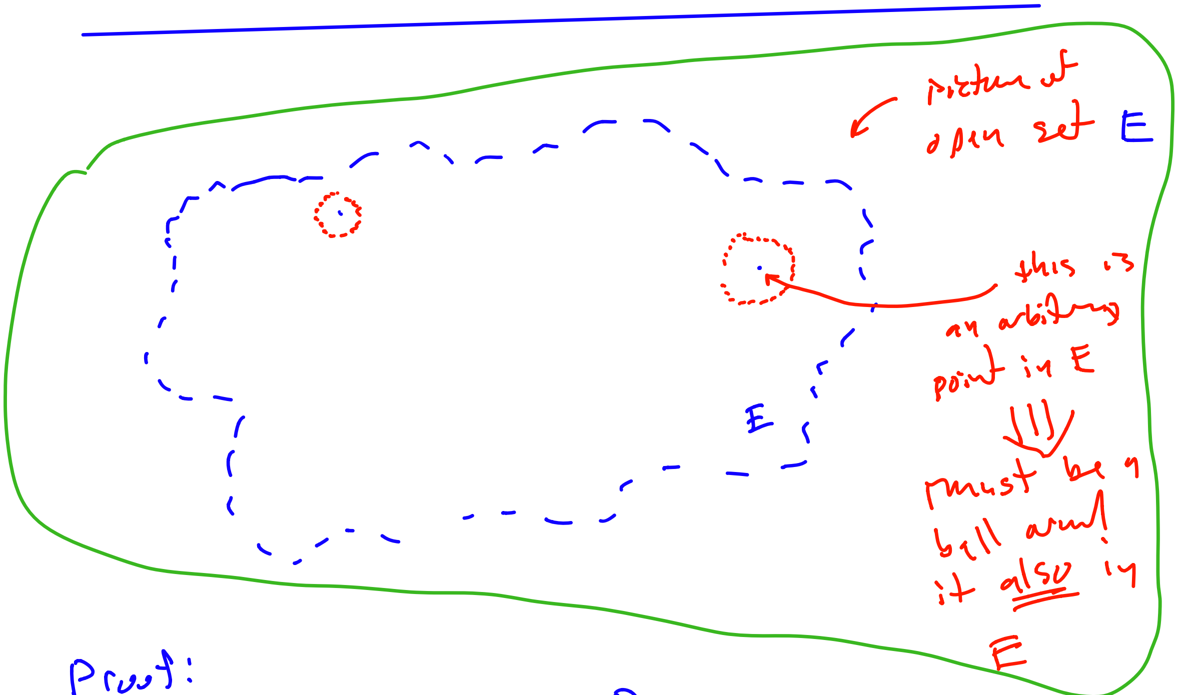
Show $E \equiv \bigcup_{\alpha \in A} O_\alpha$ is open if all the

O_α's are open



Proof:

(1) $E \equiv \bigcup_{\alpha \in A} O_\alpha$

$x \in E \implies \exists\, \alpha_x \in A \ni x \in O_{\alpha_x}$.

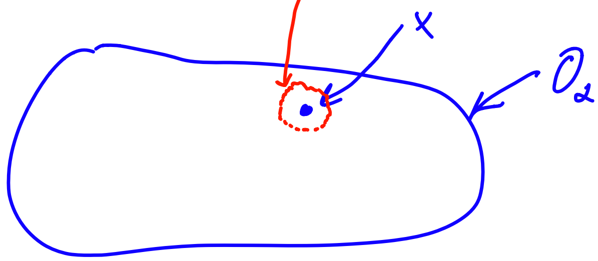
pick $\varepsilon = 1/2$

$O_x = (x-\varepsilon, x+\varepsilon)$

$= (x-\tfrac{1}{2}, x+\tfrac{1}{2})$

(2) Because O_{α_x} is open $\exists\ r>0$

$\exists$ $\underbrace{B(x,r)}$ $\subset$ O_{α_x} set inclusion



(3) $x \in B(x,r) \subset O_{\alpha_x} \subset \bigcup_{\alpha} O_\alpha = E$

$\Downarrow$

Therefore E is open

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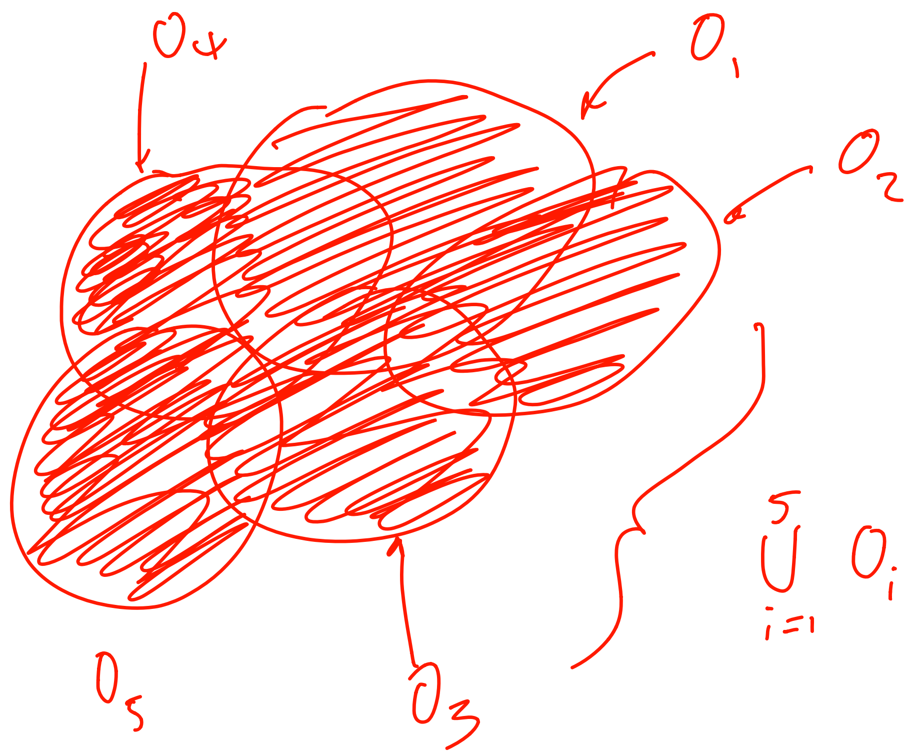
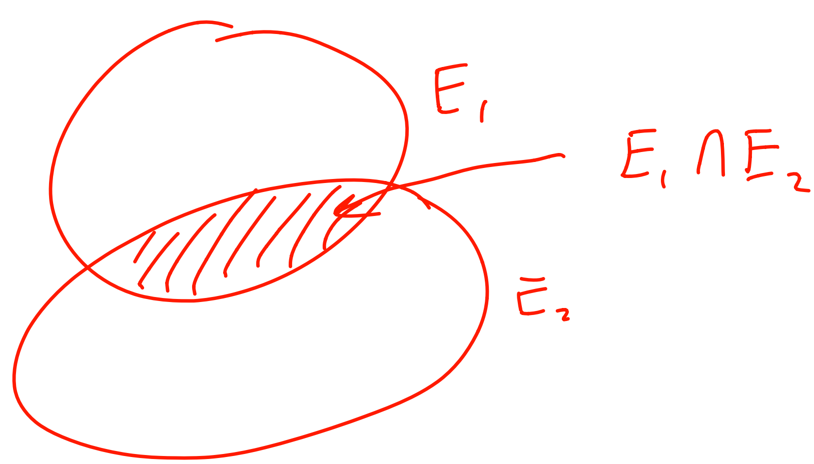
want:

$x \in E \Rightarrow \exists\ B(x,r) \subset E$

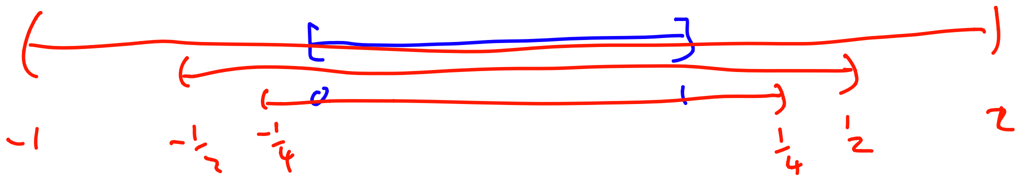
Know: $E = \bigcup_{\alpha \in A} O_\alpha$



O_4
O_1
O_2
$\bigcup_{i=1}^{5} O_i$
O_5
O_3



E_1
$E_1 \cap E_2$
E_2



$$\bigcap_{i=0}^{\infty} \left(-\frac{1}{2^i}, 1+\frac{1}{2^i}\right) = [0,1]$$

O, W, U, V

$\prod$

$$\prod_{i=1}^{\infty} \alpha_i$$

$\alpha_1 \alpha_2 \alpha_3 \alpha_4 \ldots$

(1) $\mathbb{R}$

1

2

3

4