$$\boxed{\begin{array}{c} E \subset \bigcup_{\alpha \in A} O_\alpha \Rightarrow \exists \{O_{\alpha_i}\}_{i=1}^{\infty} \subset \{O_\alpha\}_{\alpha \in A} \\ \Downarrow \\ (\text{Metric Space } M) \quad E \subset \bigcup_{i=1}^{\infty} O_{\alpha_i} \end{array}}$$

① what is separable?

answer: $\exists$ countable dense subset of M

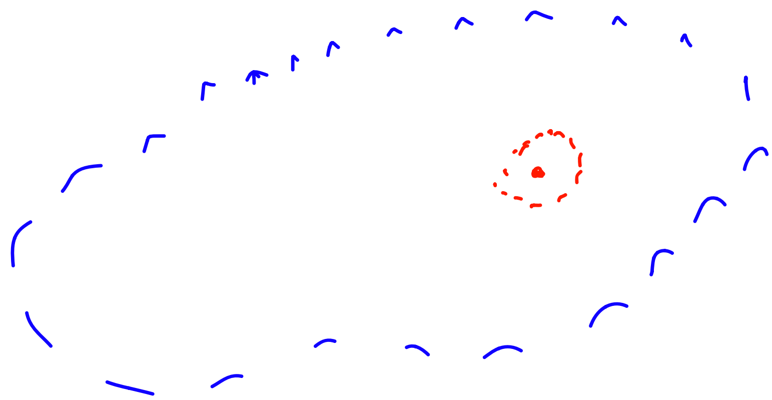
$\Rightarrow \exists \{f_i\}_{i=1}^{\infty}$ that gets as close as you want to any point $x \in M$

② $B_{i,k} = B(f_i, \frac{1}{2^k})$

$$\Downarrow$$

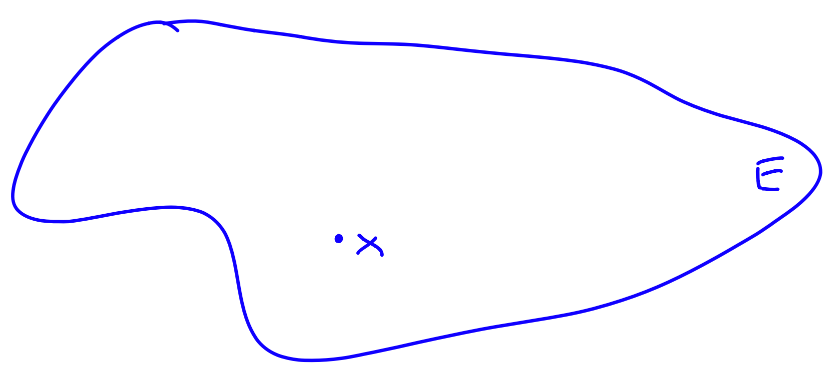
$$\{B_\ell\}_{\ell=1}^{\infty}$$

(3) Remember: the "open" picture



$x \in W \Rightarrow \exists\ \varepsilon > 0$ and
$B(x, \varepsilon) \subset W$

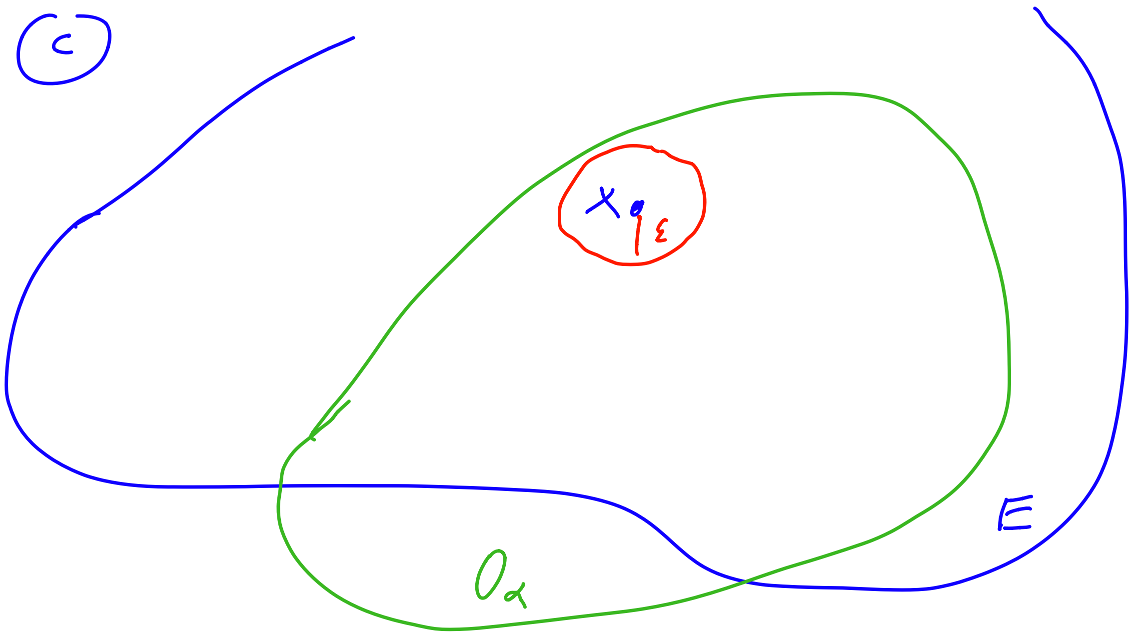
(4)



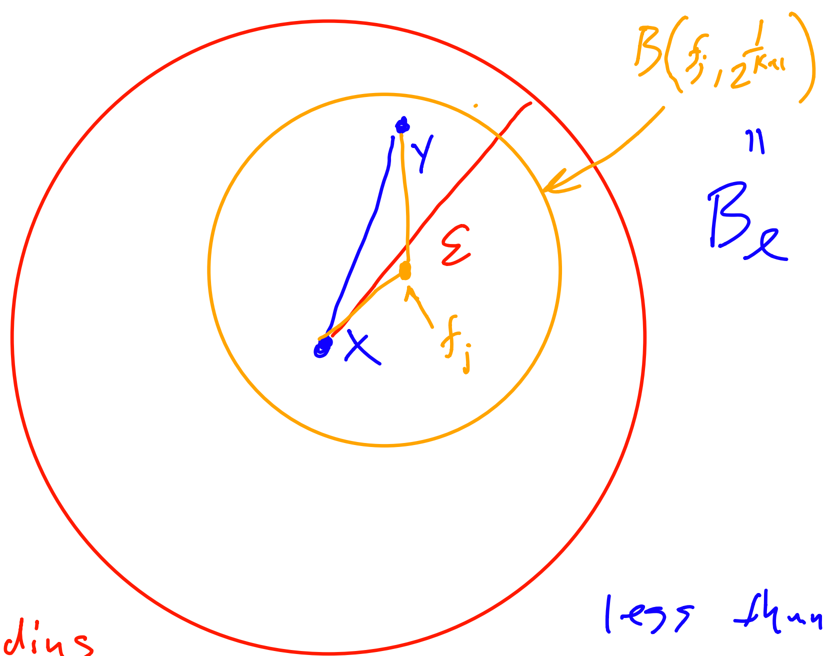
(a) $x \in O_\alpha$ for some α

(b) $\Rightarrow$ (because O_α open) that
$\exists\ \varepsilon > 0$ and $B(x, \varepsilon) \subset O_\alpha$

(c)



(d)



$\boxed{\exists \ \frac{1}{2^k} < \varepsilon}$

a ball of radius $\frac{1}{2^{k+1}}$ that includes x, stays in

less than

$\frac{1}{2^{k+1}} + \frac{1}{2^{k+1}} = \frac{1}{2^k}$

$x \in B_\ell \subset O_\alpha$

5 <u>collect everything</u>

a all $x \in E$ are in one of the B_ℓ's that is also in an O_α ... collection of all such B_ℓs is $\mathcal{B}$

$$\mathcal{B} = \{B_\ell\}_{\ell \in L}$$

$\Uparrow$

this collection is countably infinite

every $B_\ell \in \mathcal{B}$ choose an $O_\alpha \ni B_\ell$ is in O_α an name this O_α $\quad O_{\alpha_\ell}$

$$E \subset \bigcup_{\ell \in L} B_\ell = \bigcup_{B_\ell \in \mathcal{B}} B_\ell$$

$$\subset \bigcup_{\ell \in L} O_{\alpha_\ell}$$