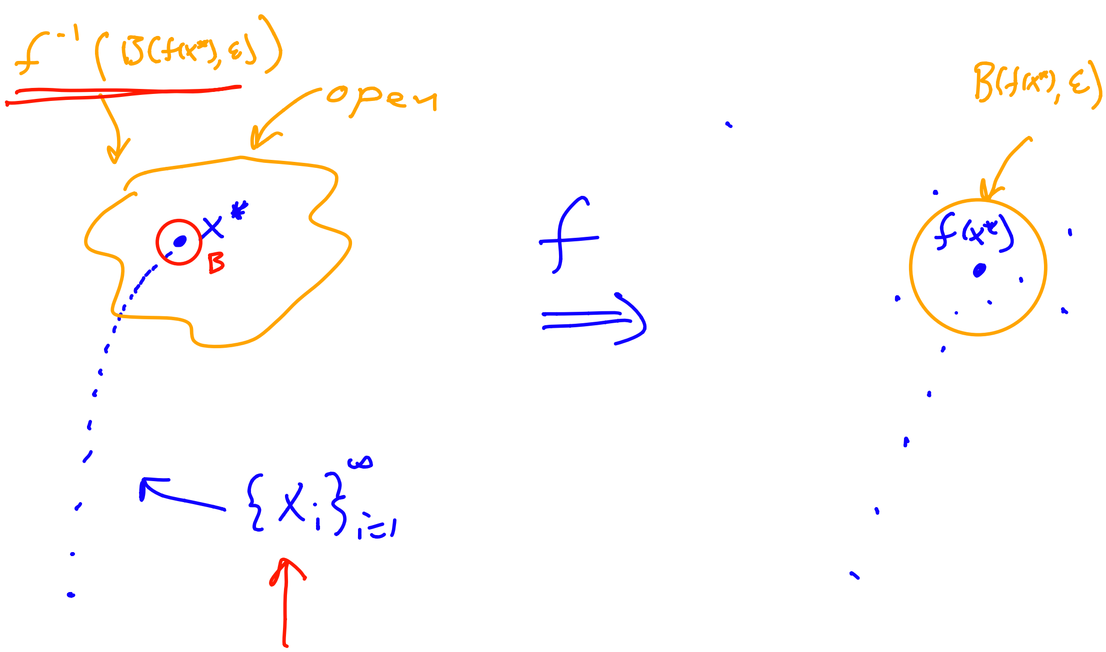
Defn I $\Longrightarrow$ Defn II:

suppose $\exists$ f satisfying I but violating II

i.e. suppose

(1) $f^{-1}(E)$ is open when E is open

(2) $\exists$ $x_i \to x^*$ and $f(x_i) \not\Longrightarrow f(x^*)$



$\Rightarrow$ $N < i$
$x_i \in B$ $\Longrightarrow$ $f(x_i) \in B(f(x^*), \varepsilon)$
$i > N$

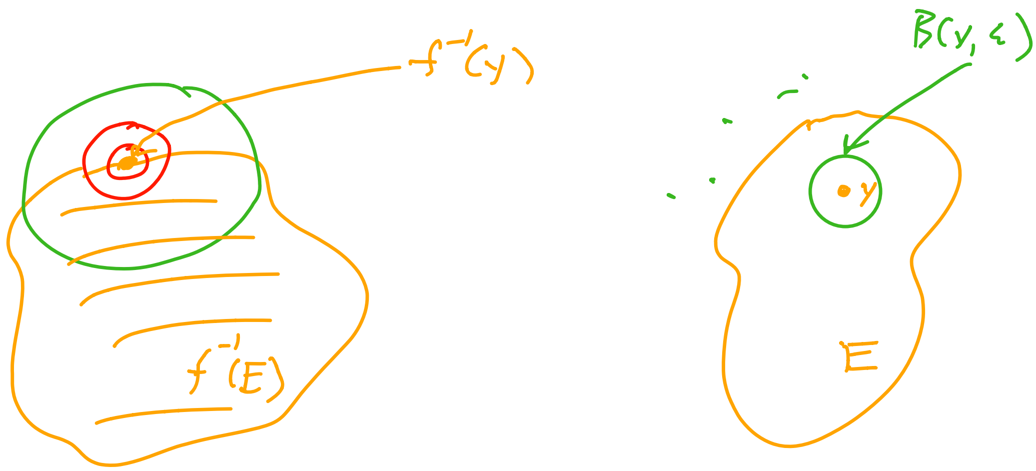
4.3.7 part 1

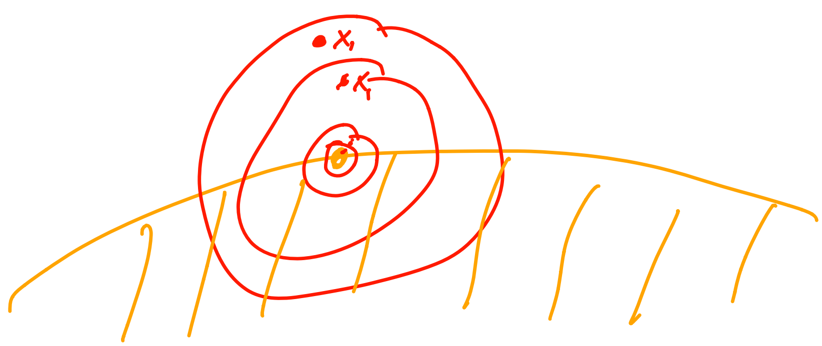
$\boxed{\text{II} \rightarrow \text{I}}$

assume

(1) $x_i \rightarrow x^* \Rightarrow f(x_i) \rightarrow f(x^*)$

(2) $\exists$ E open $\ni$ $f^{-1}(E)$ not open





$$x_1 \in (f^{-1}(E))^c \cap B_1 \qquad B(y, 1)$$

$$x_2 \in (f^{-1}(E))^c \cap B_2 \qquad B(y, \tfrac{1}{2})$$

$$x_3 \in (f^{-1}(E))^c \cap B_3 \qquad B(y, \tfrac{1}{3})$$

$$\vdots$$

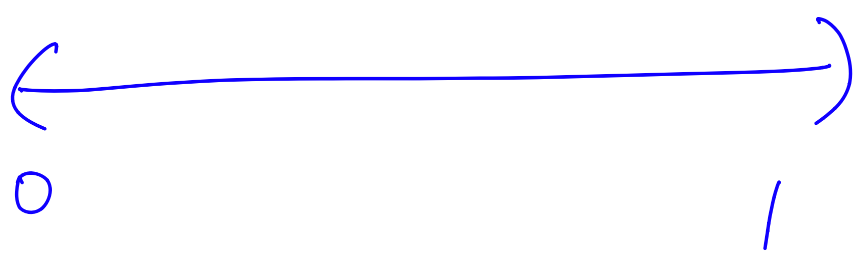
$$x_i \longrightarrow f^{-1}(y)$$

$$\rho(f(x_i), y) \geq \varepsilon$$

$$x_i \longrightarrow f^{-1}(y)$$

$$f(x_i) \not\longrightarrow f(f^{-1}(y)) = y$$

Cauchy Sequences



$$x_i = \frac{1}{i}$$

$|x_i - x_j| < \varepsilon$ given

$N_\varepsilon \le i, j$

$$\boxed{B(x_{N_\varepsilon}, x_i) < \varepsilon \qquad i > N_\varepsilon}$$

$$\mathbb{Q} \Rightarrow \mathbb{R} \subset \mathbb{C}$$