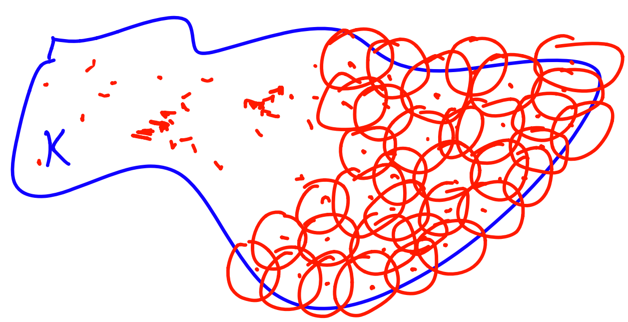
$\{x_i\}_{i=1}^{\infty} \subset K \subset X$

$\Downarrow$

there is a convergent subsequence of $\{x_i\}_{i=1}^{\infty}$

$K \Rightarrow$ totally bound

$\Rightarrow \forall \ \varepsilon > 0$ there is a finite ε-net.



1. $\varepsilon = \frac{1}{2}$

superscript indicates which ε ball center corresponds to.

one of the $B(y_i^{1/2}, \frac{1}{2})$ contains an

in finite number of those

infinite number of the x_i's : relabel

$\{x_i\}_{i=1}^{\infty}$

2. $\varepsilon = \frac{1}{4}$

repeat

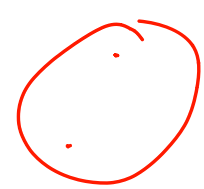
$$B(y_i^{1/4}, \tfrac{1}{4})$$

$\vdots$

→ $\hat{x}_1 \in B(y_i^{1/2}, \frac{1}{2})$

→ $\hat{x}_2 \in B(y_i^{1/4}, \frac{1}{4})$

→ $\hat{x}_3 \in B(y_i^{1/8}, \frac{1}{8})$



all but the first K they are within $\frac{1}{2^K}$ of each other $\rho(\hat{x}_i, \hat{x}_j) < \frac{1}{2^K}$

if $i, j > K$

$\{\hat{x}_i\}_{i=1}^{\infty}$ is Cauchy

$\Rightarrow \hat{x}_i \Rightarrow x^*$

$\hat{x}_i$ in K & K closed

$\Rightarrow x^* \in K$

$$\rho(\hat{x}_i, x^*) \xrightarrow[i \to \infty]{} 0$$

$\varepsilon = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \frac{1}{2^k}$

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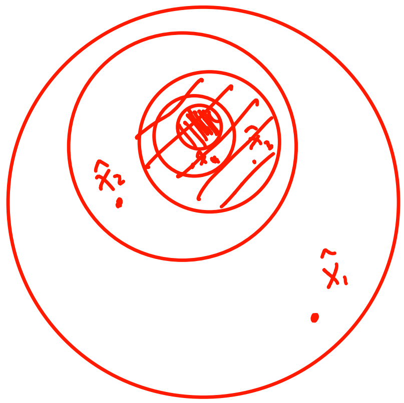
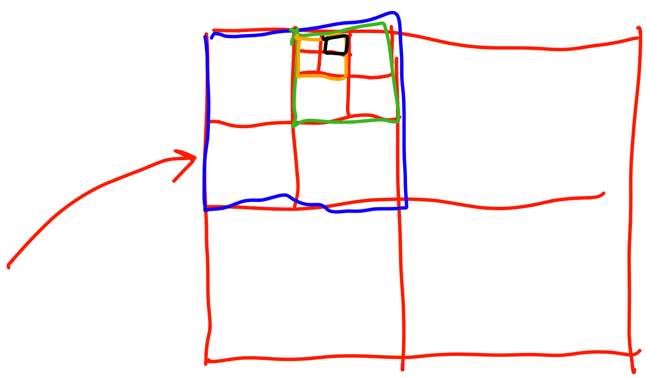
in $\mathbb{R}^n$ $\underbrace{K \text{ is compact}}_{\text{every open cover has a finite subcover}}$ $\Longleftrightarrow$ K is

① closed

② bounded

in complete metric space totally bounded





$y_1^{1/2}, \dots, y_7^{1/2}$