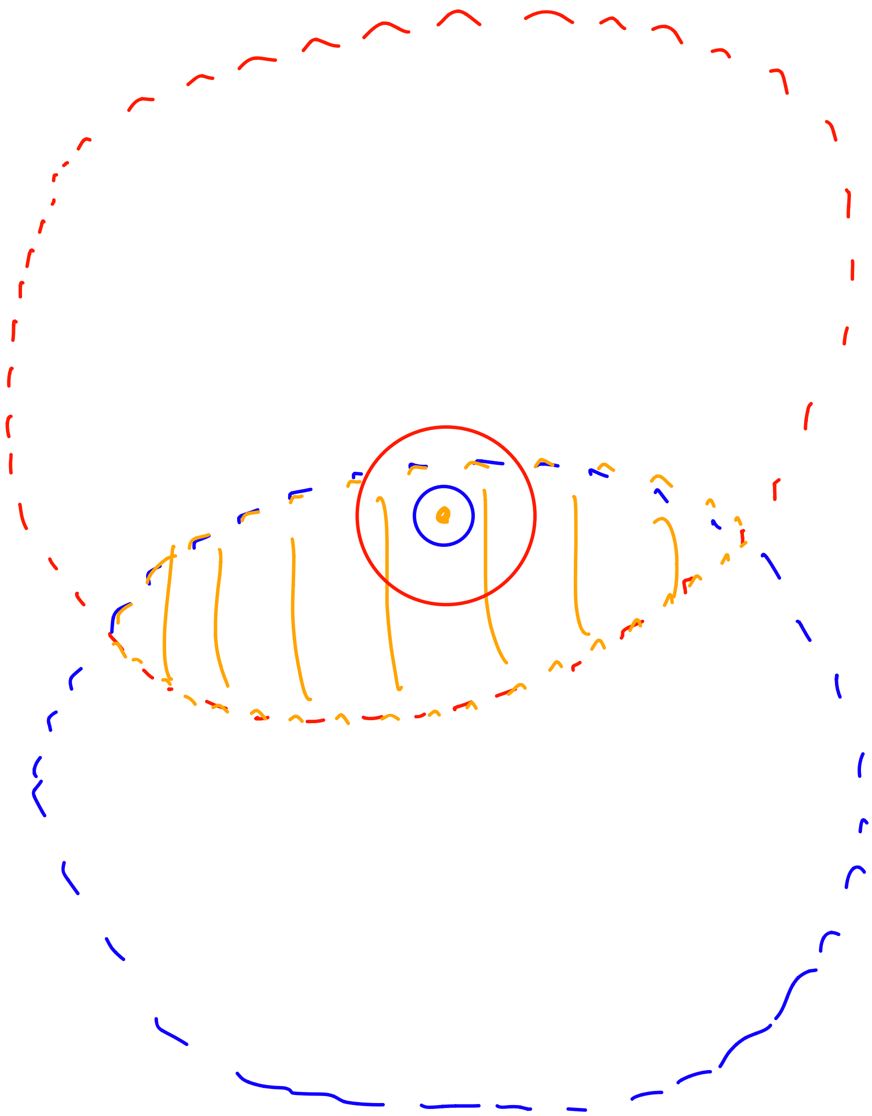
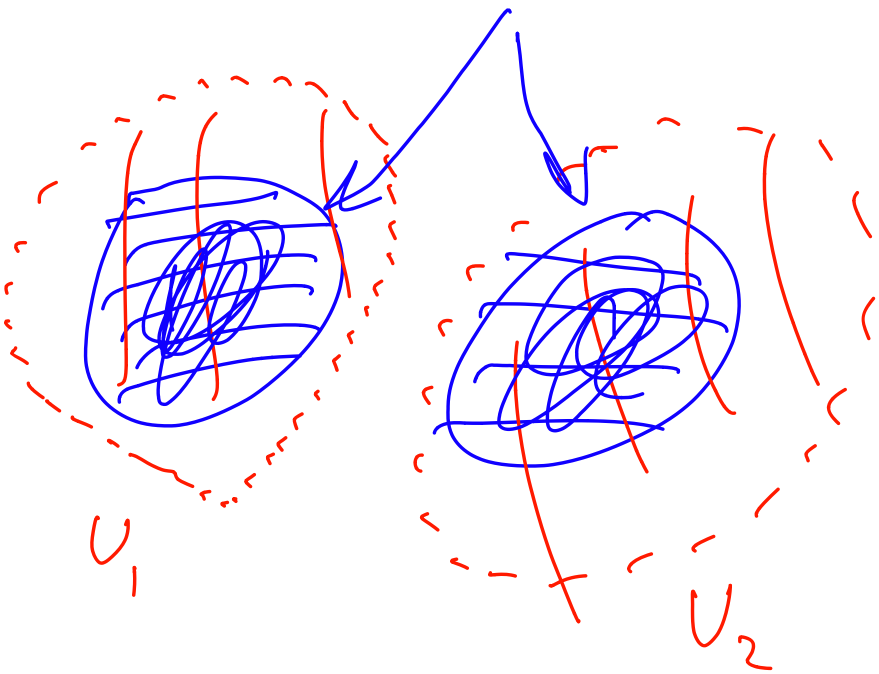
<u>Prove:</u> if O_1 and O_2 are open so is $O_1 \cap O_2$



D is disconnected



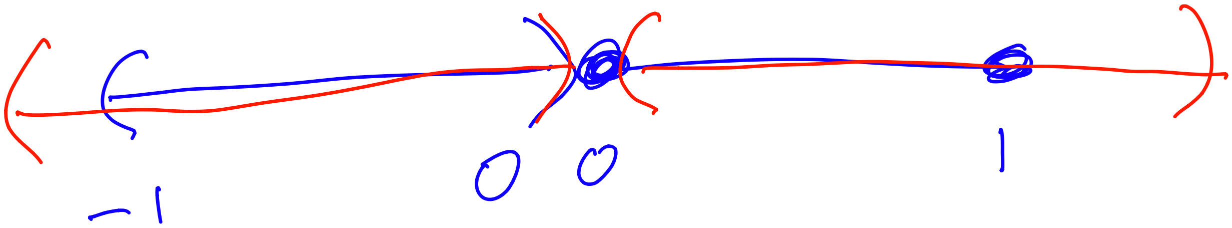
U_1 & U_2 are open

$U_1 \cap U_2 = \phi$

$D \cap U_1 \neq \phi$

$D \cap U_2 \neq \phi$

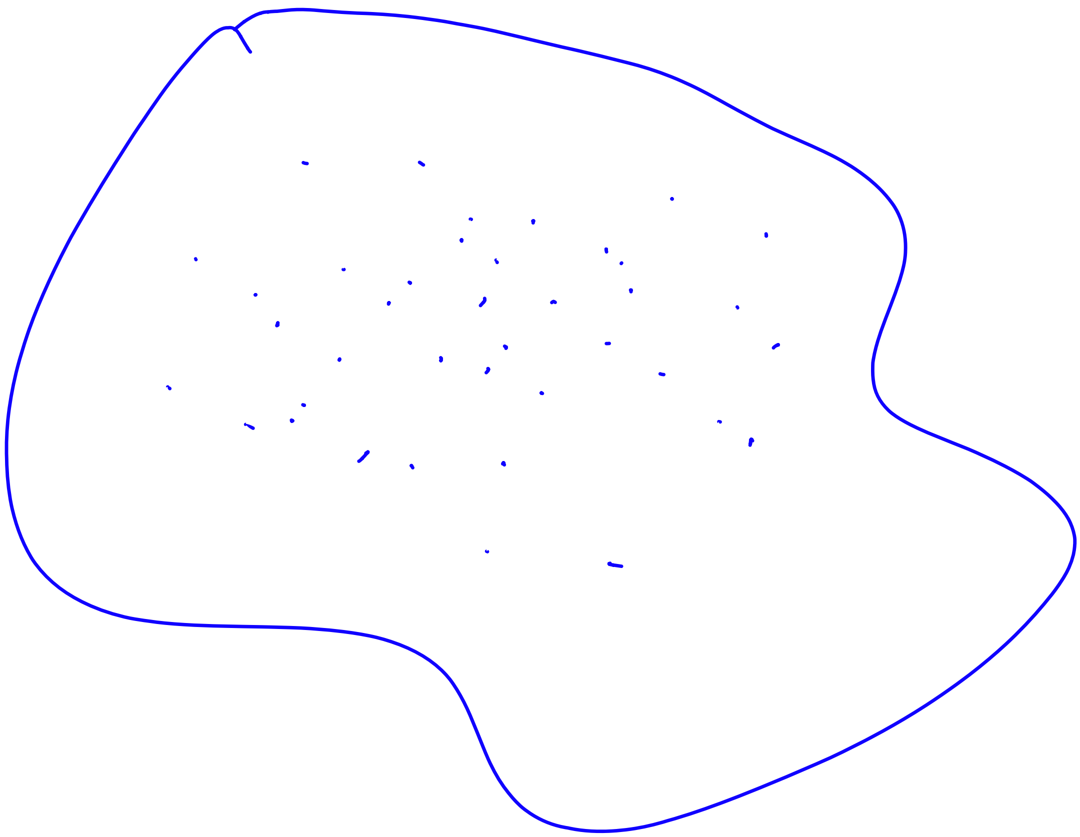
$D \subset U_1 \cup U_2$

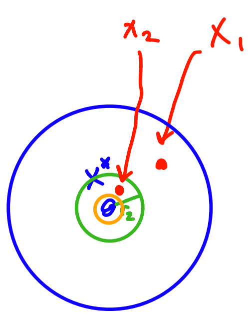


$(-1, 0) \quad [0, 1]$

disconnected $\neq$ disjoint

disconnected $\subset$ disjoint





1

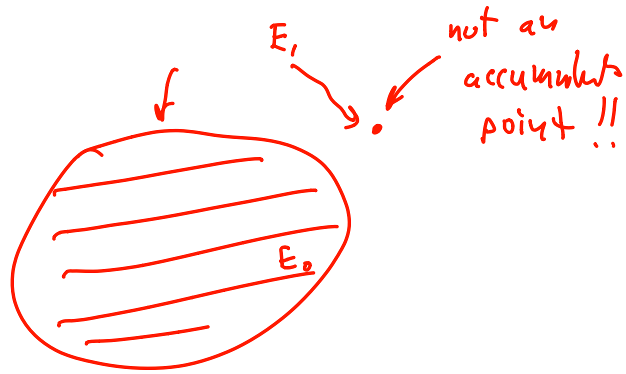
$$r_2 = \frac{\rho(x^*, x_1)}{2}$$

$x_1 \quad x_2 \quad x_5 \longrightarrow x^*$

x^* is an accumulation point of E

if $\forall \varepsilon > 0 \ \exists \ x \in E \ni$

1. $x \neq x^*$
2. $\rho(x, x^*) < \varepsilon$



$$E \equiv E_0 \cup E_1$$

$\rightarrow$ E_1 <u>not</u> an accumulation point of E.

$x^* \in E$ is isolated if $\exists$

$\varepsilon > 0 \ni$

$$B(x^*, \varepsilon) \cap E = x^*$$

Cauchy sequences

① $x_1 \ x_2 \ x_3 \ x_4 \ x_5$

② tails of sequences

$$x_{N+1}, x_{N+2}, x_{N+3}, x_{N+4}, \ldots$$

③ $B(y, \varepsilon)$ ... of $\rho(y, x) < \varepsilon$

$\{x_i\}_{i=1}^{\infty}$ is <u>Cauchy</u> if for any $\varepsilon > 0$

ⓐ $\rho(x_i, x_j) < \varepsilon$ for

x_i, x_j in <u>some</u> tail

ⓑ $\exists \ N_\varepsilon$ such that

$x_j \in B(x_i, \varepsilon) \ \forall \ i, j > N_\varepsilon$

$\overline{B}(x_{i_1}, \frac{1}{2})$ $\quad N_1 \quad i_1 > N_1$

$\overline{B}(x_{i_2}, \frac{1}{2^2})$ $\quad N_2 \quad i_2 > N_2$

$\overline{B}(x_{i_3}, \frac{1}{2^3})$ $\quad N_3 \quad i_3 > N_3$

$$F_K = \bigcap_{j=1}^{K} \overline{B}\left(x_{i_j}, \frac{1}{2^j}\right)$$

(a) $F_1 \supset F_2 \supset F_3 \ldots$

(b) $\operatorname{diam} F_i = \frac{1}{2^{i-1}}$

(c) $D \equiv \bigcap_{K=1}^{\infty} F_K \neq \emptyset$

Prove (c) and that D is a single point.

Extra credit.