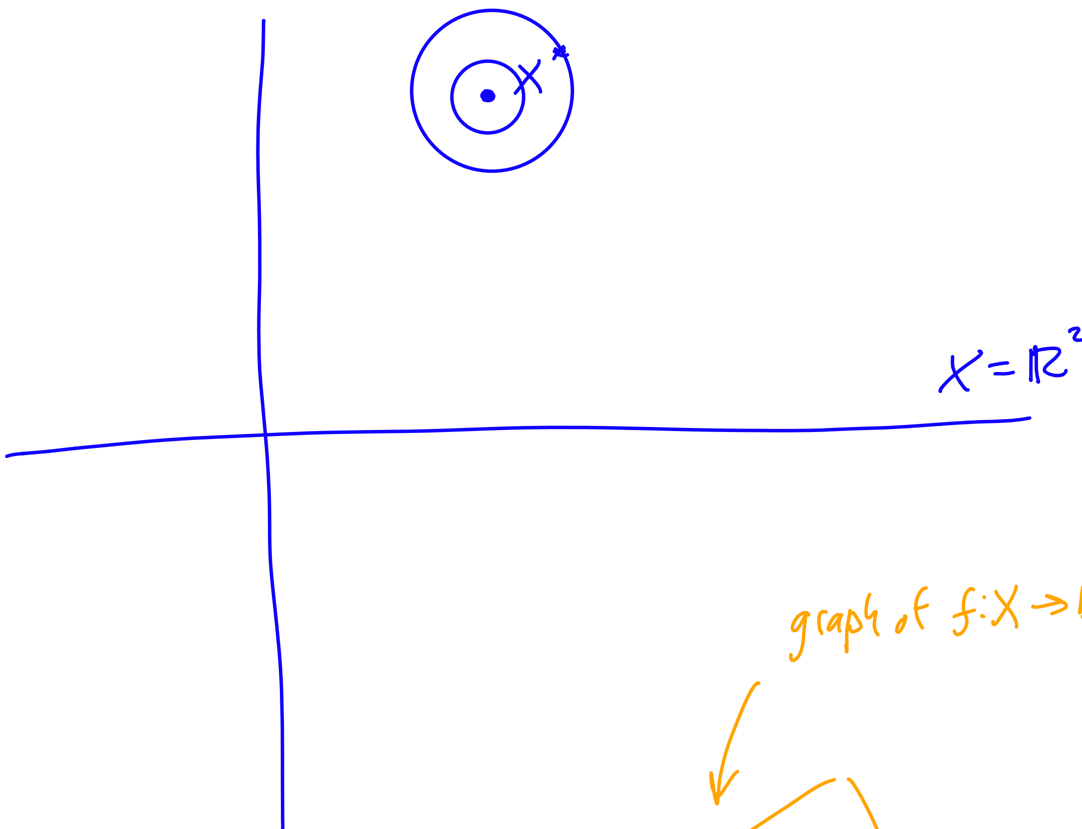
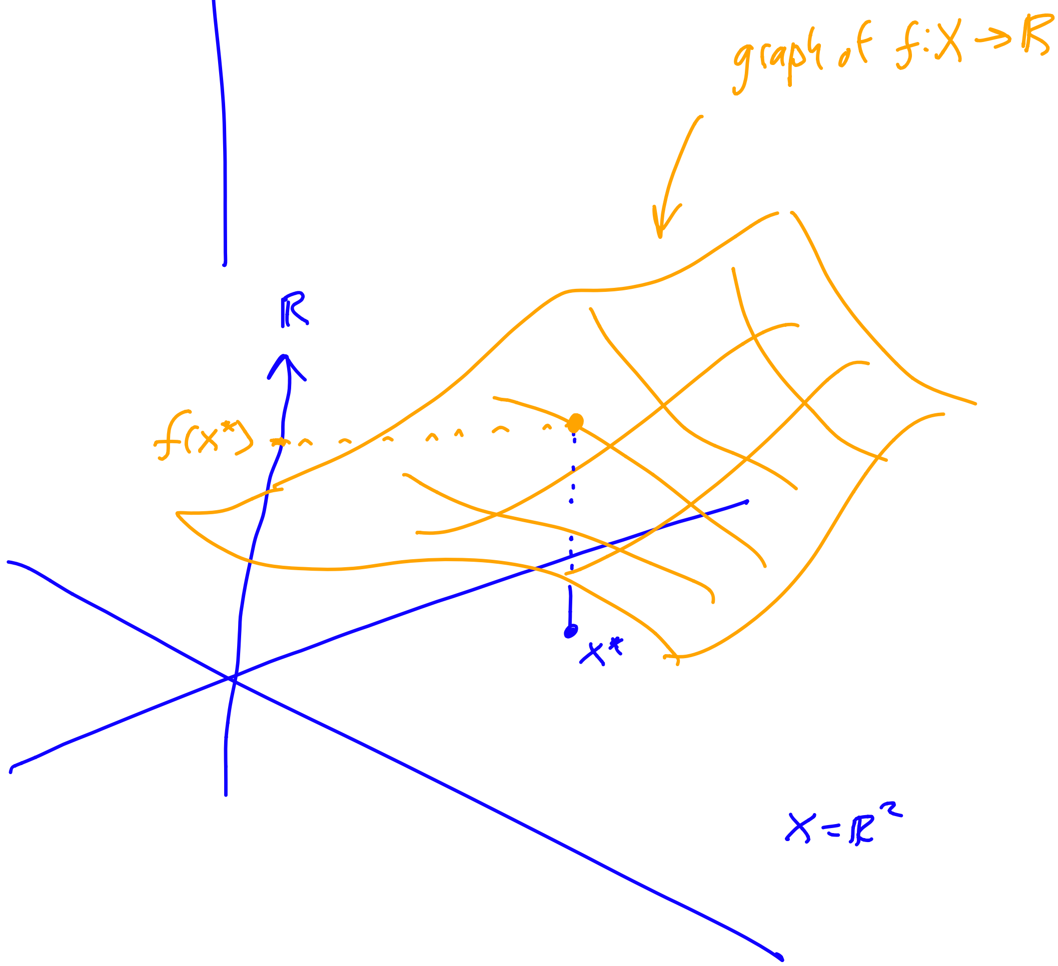
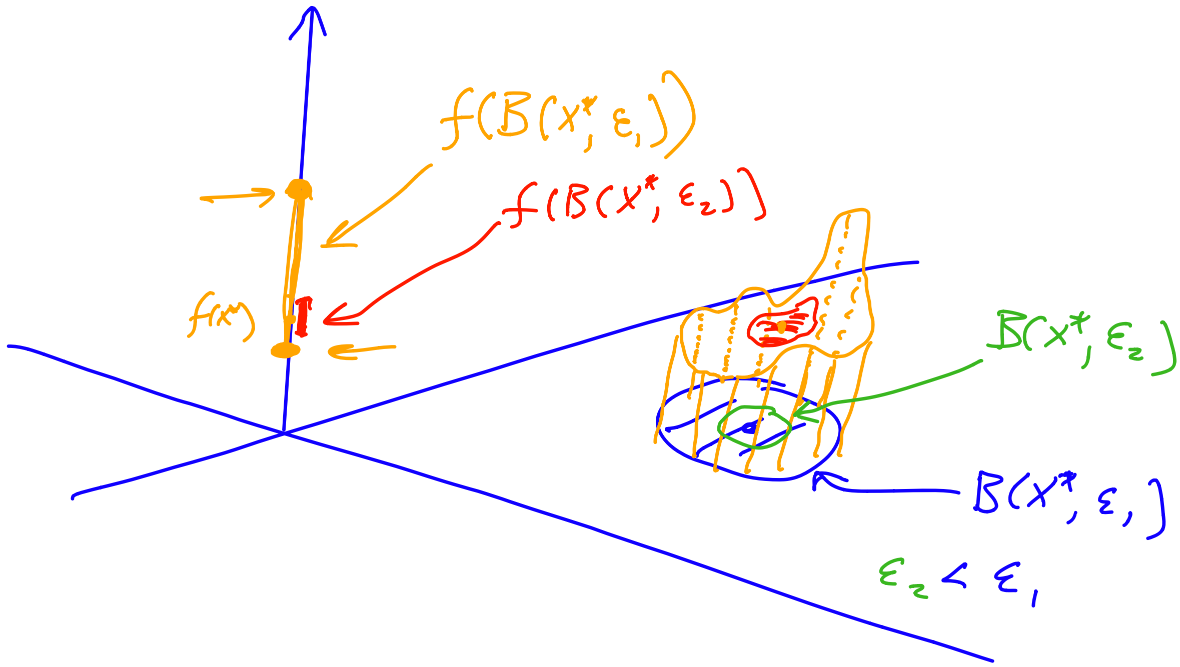


x*
X = ℝ²



graph of f: X → ℝ
ℝ
f(x*)
x*
X = ℝ²



$$\boxed{\begin{aligned} \liminf_{x \to x^*} f(x) &= \lim_{\varepsilon \to 0} \inf_{B(x^*, \varepsilon)} f(x) \\ \limsup_{x \to x^*} f(x) &= \lim_{\varepsilon \to 0} \sup_{B(x^*, \varepsilon)} f(x) \end{aligned}}$$

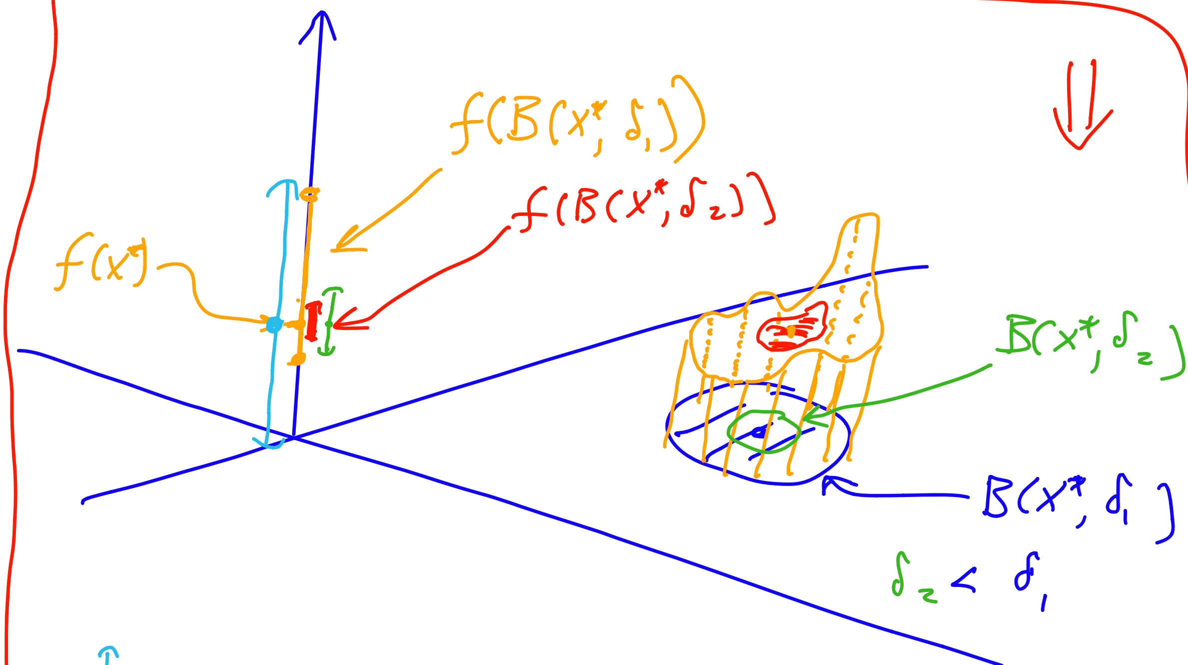
f is continuous at $x = x^*$

$\Updownarrow$ $\Downarrow$ $\Uparrow$

$$\liminf_{x \to x^*} f(x) = \limsup_{x \to x^*} f(x)$$

use: Def 4.3.7

Fig 19 section 4.3



$= B(f(x^*), \varepsilon_1) \supset f(B(x^*, \delta_1))$

$= B(f(x^*), \varepsilon_2) \supset f(B(x^*, \delta_2))$

This picture shows the Def III of continuity

$\Rightarrow \liminf\limits_{x \to x^*} f(x) = \limsup\limits_{x \to x^*} f(x)$

$\Uparrow$

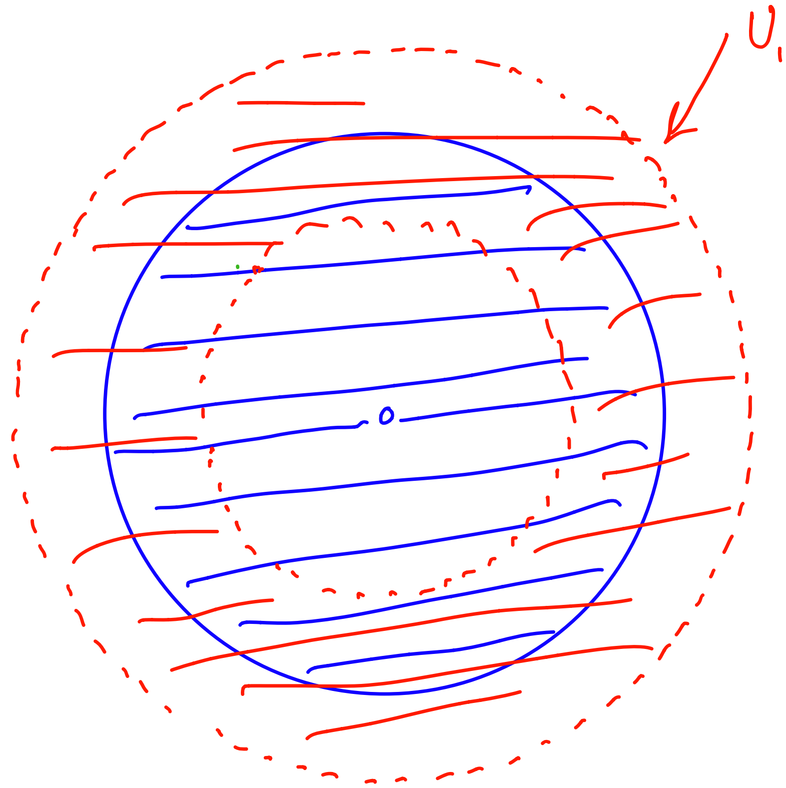
given $\varepsilon > 0$ $\exists$

$\Rightarrow$ δ small enough that

$$\sup_{B(x^*,\delta)} f(x) - \inf_{B(x^*,\delta)} f(x) < \varepsilon$$

but this implies Def III of continuity is true at x^*.

4.7.7



Hint: look at the open set U_1 (an open annulus)

$$D = \{ x \mid 0 < |x| \leq 1 \}$$

norm of x

$$= \sqrt{x \cdot x}$$

$$= \sqrt{x_1^2 + x_2^2}$$

$$U_1 = \{ x \mid 0 < r_1 < |x| < r_2 \}$$

$$r_1 = \tfrac{1}{2}, \quad r_2 = 1.25$$

$$= \{ x \mid \tfrac{1}{2} < |x| < 1.25 \}$$