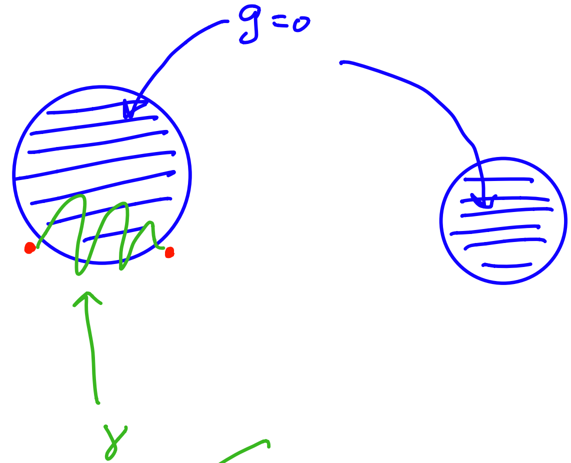
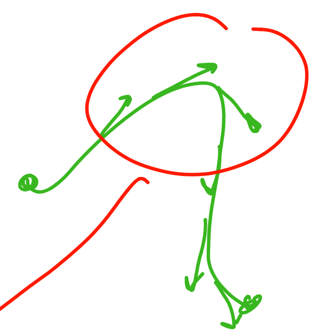
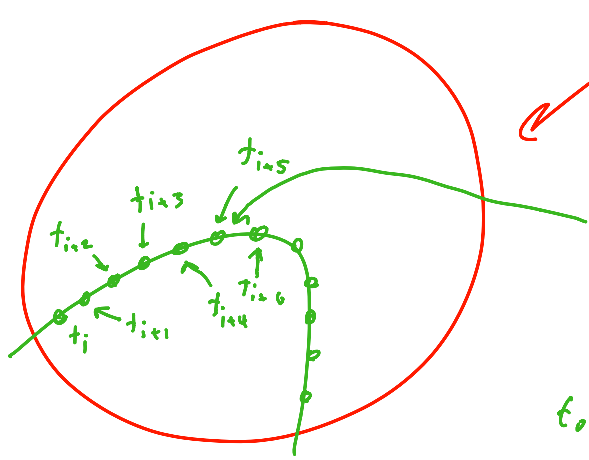
Discussion of Exercise 4.7.10

$g = 1$



$$\text{Length}(\gamma) \quad \underbrace{\int_0^1 g(\gamma(t))\,|\dot\gamma(t)|\,dt}_{\displaystyle\int_0^1 |\dot\gamma(t)|\,dt}$$





$$\text{length} = (t_{i+6} - t_{i+5})\,|\dot\gamma(t)|$$
$$= \Delta t\,|\dot\gamma(t_{i+5})|$$

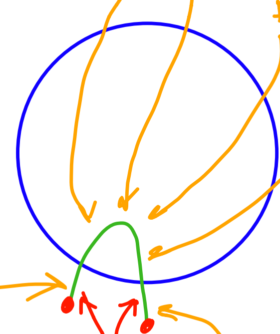
$t_0 = 0$, t_1, t_2 ... $t_n = 1$

$$\underline{\text{length}(\gamma)} \approx \sum_{i=0}^{n-1} \underline{|\gamma(t_i)|\,\Delta t}$$

$$\Big\Downarrow \quad \Delta t \to 0$$

$$\int_0^1 |\gamma(t)|\,dt$$

I am "charged" nothing for this length because $g=0$ inside



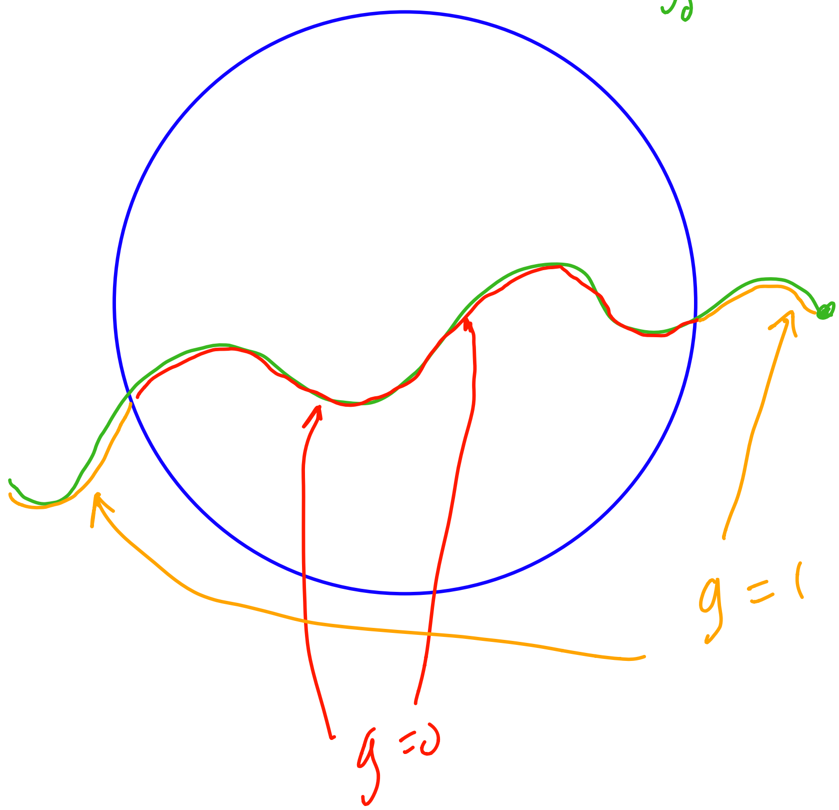
$$\int_0^1 g(\gamma(t))\,|\dot{\gamma}(t)|\,dt$$

these two pieces I am "charged" the usual Euclidean length because $g=1$ outside

conclusion: these two pieces add their lengths to get total l_g for γ

$$\int_0^1 |\dot{\gamma}(t)|\, dt$$

$$\int_0^1 g(\gamma(t))\,|\dot{\gamma}(t)|\, dt$$



convince yourself that any minimal path will be a piece of a straight line outside of the disks.

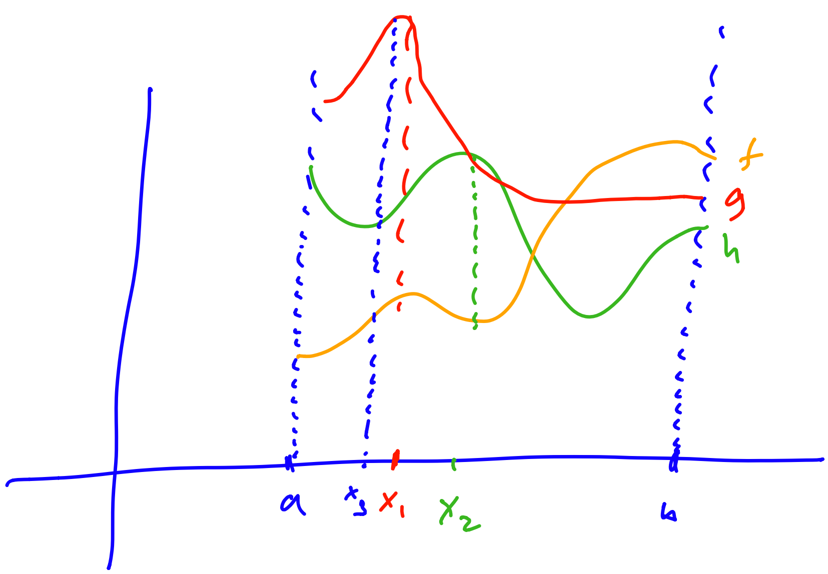
Chatty about
4.7.14

*) $\rho(x, y) = \rho(y, x)$

*) $\rho(x, y) = 0 \iff x = y$

*) $\rho(x, z) \leq \rho(x, y) + \rho(y, z)$

$$\rho(f, g) \equiv \max_{x \in [a,b]} |f(x) - g(x)|$$



$$\rho(f,h) \le \rho(f,g) + \rho(g,h)$$

$$\rho(f,h) = |f(x_2) - h(x_2)| = |f(x_2) - g(x_2) + g(x_2) - h(x_2)|$$

$$\le |f(x_2) - g(x_2)| + |g(x_2) - h(x_2)|$$

get x_2 from the theorem that says continuous functions attain their mins and max's on compact sets

$$\le \max_x |f(x) - g(x)| = \rho(f,g)$$

$$\le \max_x |g - h| = \rho(g,h)$$